6.29) At $t=0^-$: $\quad i_L(0^-) = \frac{18-12}{6+6} = 0.5\,A$

$$V_c(0^-) = 6 \times 0.5 + 12 = 15\,V$$

At $t=0$: $\quad V_c(0) = V_c(0^-) = 15$

$$i_c(0) = -i_L(0) = -i_L(0^-) = -0.5\,A$$

$$\dot{V}_c(0) = \frac{i_c(0)}{C} = -\frac{0.5}{C}$$

At $t>0$:

$$\alpha = \frac{R}{2L} = \frac{6+6}{2 \times 0.1} = 60\ Np/s$$

In order to have a critically damped circuit response

$$\alpha = \omega = \frac{1}{\sqrt{LC}} = 60$$

$$\Rightarrow C = \frac{1}{360}\,F$$

$$\therefore \quad \dot{V}_c(0) = -\frac{0.5}{C} = -0.5 \times 360 = -180\ V/s$$

$$\therefore \quad V_c(t) = V_c(\infty) + (B_1 + B_2 t)e^{-at}$$

$$= 12 + (B_1 + B_2 t)e^{-at}$$

where $B_1 = V_c(0) - V_c(\infty) = 15 - 12 = 3V$

$$B_2 = \dot{V}_c(0) + \alpha[V_c(0) - V_c(\infty)]$$

$$= -180 + 60[15-12]$$

$$= 0$$

$\therefore V_c(t) = 12 + 3e^{-60t}$, for $t \geq 0$

6.35) At $t=0^-$: $V_c(0^-) = 8$

$i_L(0^-) = 0$

At $t \geq 0$: $V(\infty) = \frac{4}{3} V$

$i_c(0) = i_L(0) = i_L(0^-) = 0$

$$\alpha = \frac{R}{2L} = \frac{2/3}{2 \times 0.5 \times 10^{-3}} = 666.7 \text{ Np/s}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 10^{-3} \times 2 \times 10^{-3}}} = 1000 \text{ rad/s}$$

Since $\alpha < \omega_0$, the response is underdamped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 745.4 \text{ rad/s}$$

$$D_1 = V_c(0) - V_c(\infty) = 8 - \frac{4}{3} = \frac{20}{3}$$

$$D_2 = \frac{\frac{1}{C} i_c(0) + \alpha [V_c(0) - V_c(\infty)]}{\omega_d} = 5.96$$

$$V_c(t) = V_c(\infty) + [D_1 \cos(\omega_d t) + D_2 \sin(\omega_d t)] e^{-\alpha t}$$

$$= \frac{4}{3} + \left[\frac{20}{3} \cos(745.4t) + 5.96 \sin(745.4t)\right] e^{-666.7t},$$

for $t \geq 0$

6.38 $t=0^-$ $i_L(0^-) = 1.5mA$

$v_c(0^-) = 0$

$t=0^+$ $i_L(0^+) = i_L(0^-) = 1.5mA$

$v_c(0^+) = v_c(0^-) = 0 = V_L(0^+) = L \frac{di_L}{dt}\Big|_{t=0^+}$

$\Rightarrow i_L'(0^+) = 0$

$t=\infty$ $i_L(\infty) = 1.5mA$

$\Rightarrow$ i_L keeps same value when $t=0$ and $t=\infty$, and since $i_L'(0^+)=0$, ~~i~~ i_L doesn't change when $t=0$.

$\Rightarrow i_L(t) = 1.5mA$

6.50. $V'' + 5V' + 6V = 144$

Solve this ODE $\Rightarrow$ $V(t) = C_1 e^{-3t} + C_2 e^{-2t} + C_3$, C_1, C_2, C_3 are constants

$\because V(0) = 16V$ $\therefore$ $C_1 + C_2 + C_3 = 16$ ①

$\because V'(0) = 9.6V/s$ $\therefore$ $V'(t)\big|_{t=0} = -3C_1 e^{-3t} - 2C_2 e^{-2t}\big|_{t=0} = -3C_1 - 2C_2 = 9.6$ ②

$\because V(0) = 16V,\ V'(0) = 9.6V/s$ $\therefore$ $\cancel{V(t)}$ $V''(0) + 5V'(0) + 6V(0) = 144$

$\Rightarrow V''(0) + 5 \times 9.6 + 6 \times 16 = 144$

$\Rightarrow \cancel{V''(t)}\ V''(0) = V''(t)\big|_{t=0} = 0$

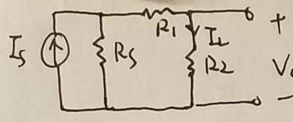
$\Rightarrow 9C_1 e^{-3t} + 4C_2 e^{-2t}\big|_{t=0} = 0$

$\Rightarrow 9C_1 + 4C_2 = 0$ ③

$$\therefore \begin{cases} C_1 + C_2 + C_3 = 16 & ① \\ -3C_1 - 2C_2 = 9.6 & ② \\ 9C_1 + 4C_2 = 0 & ③ \end{cases} \Rightarrow \begin{cases} C_1 = 6.4 \\ C_2 = -14.4 \\ C_3 = 24 \end{cases}$$

$\therefore V(t) = (6.4e^{-3t} - 14.4e^{-2t} + 24)\ V$

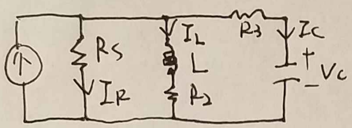
6.54 1) $t=0^-$ ($t<0$)



$I_L(t=0^-) = I_S \cdot \frac{R_S}{R_S+R_1+R_2} = 0.1A$

$V_C(t=0^-) = I_L(t=0^-) \cdot R_2 = 2V$

2) $t=0^+$



$I_L(t=0^+) = I_L(t=0^-) = 0.1A$

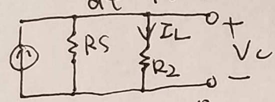
$V_C(t=0^+) = V_C(t=0^-) = 2V$

$$\begin{cases} KCL: I_S = I_R(t=0^+) + I_C(t=0^+) + I_L(t=0^+) \\ KVL: I_C(t=0^+) \cdot R_3 + V_C(t=0^+) = I_R(t=0^+) \cdot R_3 \end{cases}$$

$\Rightarrow I_C(t=0^+) = C\frac{dV_C}{dt}\Big|_{t=0^+} = 0.02A$

$\frac{dV_C}{dt}\Big|_{t=0^+} = 4\ V/s$

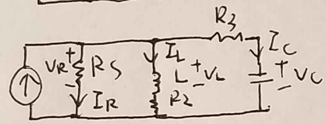
3) $t=\infty$



$I_L(t=\infty) = I_S \cdot \frac{R_S}{R_S+R_2} = 0.12A$

$V_C(t=\infty) = I_L(t=\infty) \cdot R_2 = 2.4V$

4) $t>0$



KCL: $I_S = I_R + I_L + I_C$ ①

Using $\begin{cases} I_R = \frac{V_R}{R_S} = \frac{V_C + I_C R_3}{R_S} & ② \\ I_C = C\frac{dV_C}{dt} & ③ \end{cases}$ into ①, leads to $I_S = I_L + C\frac{dV_C}{dt} + \frac{V_C + C\frac{dV_C}{dt} \cdot R_3}{R_S}$ ④

$\Rightarrow I_L = I_S - C\frac{dV_C}{dt} - \frac{V_C}{R_S} - C\frac{R_3}{R_S}\frac{dV_C}{dt}$ ⑤

$\frac{dI_L}{dt} = -\frac{C(R_3+R_S)}{R_S}\frac{d^2V_C}{dt^2} - \frac{1}{R_S}\frac{dV_C}{dt}$ ⑥

KVL: $V_L + I_L \cdot R_2 = V_C + I_C R_3$ ⑦

Using ③ and $V_L = L\frac{dI_L}{dt}$ ⑧ into ⑦, leads to $L\frac{dI_L}{dt} + I_L R_2 = V_C + R_3 C\frac{dV_C}{dt}$ ⑨

Using ⑤ and ⑥ into ⑨, leads $\frac{d^2V_C}{dt^2} + 12\frac{dV_C}{dt} + 50V_C = 120$

and plugging ~~the~~ numbers into ⑨

$\alpha = \frac{12}{2} = 6$, $\omega_0 = \sqrt{50} = 5\sqrt{2} \approx 7.07 \Rightarrow \alpha < \omega_0$, underdamped

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{14} \approx 3.74$

$D_1 = V_C(0) - V_C(\infty) = -0.4$ $\quad D_2 = \frac{V_C'(0) + \alpha[V_C(0) - V_C(\infty)]}{\omega_d} = \frac{4 - 6 \times 0.4}{\sqrt{14}} = \frac{4\sqrt{14}}{35} \approx 0.428$

At $t>0$

$\Rightarrow V_C(t) = V_C(\infty) + e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t)$

$= 2.4 + e^{-6t}\left(-0.4\cos\sqrt{14}t + \frac{4\sqrt{14}}{35}\sin\sqrt{14}t\right)$ V

7.1 $v(t) = -4\sin(8\pi\times10^3 t - 45^\circ)\ V$

$= -4\cos(8\pi\times10^3 t - 45^\circ - 90^\circ)\ V$

$= 4\cos(8\pi\times10^3 t - 45^\circ - 90^\circ + 180^\circ)\ V$

$= 4\cos(8\pi\times10^3 t + 45^\circ)\ V$

$= A\cos(2\pi f t + \phi)\ V$

$\therefore A = 4\ V$

$f = 4\ KHZ$

$\phi = 45^\circ$

$T = \frac{1}{f} = 2.5\times10^{-4}\ s$

7.22

1).

$$V_1(t) = 4\cos(377t - 30^\circ)$$

$$V_1 = 4e^{-j30^\circ}\ V$$

2).

$$V_2(t) = -2\sin(8\pi \times 10^4 t + 18^\circ)\ V$$

$$= 2\cos(8\pi \times 10^4 t + 18^\circ + 90^\circ)$$

$$V_2 = 2e^{j108^\circ}\ V$$

3).

$$V_3(t) = 3\sin(1000t + 53^\circ) - 4\cos(1000t - 17^\circ)$$

$$= 3\cos(1000t + 53^\circ - 90^\circ) + 4\cos(1000t - 17^\circ + 180^\circ)$$

$$V_3 = 3e^{-j37^\circ} + 4e^{j163^\circ}$$

$$= 3\cos 37^\circ - j3\sin 37^\circ + 4\cos 163^\circ + j4\sin 163^\circ$$

$$= 2.4 - j1.81 - 3.83 + j1.17$$

$$= -1.43 - j0.64$$

$$= 1.57e^{-j155.9^\circ}$$

7.10) Not: $z = a + jb \Rightarrow r = \sqrt{a^2 + b^2}$, $\theta = \begin{cases} \tan^{-1}\frac{b}{a} & \text{for } a > 0 \\ \tan^{-1}\frac{b}{a} + 180° & \text{for } a < 0 \end{cases}$

a) $z_1 = 3 + j4 = 5 \underline{\angle 53.13°}$

$r = \sqrt{3^2 + 4^2} = 5$, $\theta = \tan^{-1}\frac{4}{3} = 53.13°$

$\Rightarrow$ polar form: $z_1 = 5(\cos(53.13°) + j\sin(53.13°)) = 5 \underline{\angle 53.13°}$

b) $z_2 = -6 + j8 = 10 \underline{\angle 126.87°}$

$r = \sqrt{(-6)^2 + 8^2} = 10$, $\theta = \tan^{-1}\frac{8}{-6} + 180° = 126.87°$

polar form: $z_2 = 10(\cos(126.87°) + j\sin(126.87°)) = 10 \underline{\angle 126.87°}$

c) $z_3 = -6 - j4 = 7.21 \underline{\angle -146.31°}$ or $7.21 \underline{\angle 213.69°}$

$r = \sqrt{(-6)^2 + (-4)^2} = 7.21$, $\theta = \tan^{-1}\frac{-4}{-6} + 180° = -146.31°$

d) $z_4 = j2 = 2 \angle 90°$

$r = \sqrt{0^2 + 2^2} = 2$, $\theta = \tan^{-1}\frac{2}{0} = \tan^{-1}\infty = 90°$

e) $z_5 = (2 + j)^2 = 4 + j2 + j^2 = 4 + j2 - 1 = 3 + j2$

$r = \sqrt{3^2 + 2^2} = 3.61$, $\theta = \tan^{-1}\frac{2}{3} = 33.69°$

$\Rightarrow z_5 = 3.61 \underline{\angle 33.69°}$

f) $z_6 = (3-j2)^3 = 27 - 3(9)(j2) + 3(3)(j2)^2 - (j2)^3$

$= 27 - j54 - 36 + j8$

$= -9 - j46$

$r = \sqrt{(-9)^2 + (-46)^2} = 46.87 \quad \theta = \tan^{-1}\frac{-46}{-9} + 180° = -101.07°$

$\Rightarrow z_6 = 46.87 \underline{/-101.07°}$

g) $z_7 = (-1+j)^{1/2} \Rightarrow z_7^2 = (-1+j)$

for z_7^2 : $r = \sqrt{(-1)^2 + 1^2} = 1.41$, $\theta = \tan^{-1}\frac{1}{-1} + 180° = 135°$

If we multiply two complex numbers in polar form

we get : $z_1 z_2 = r_1 \times r_2 \underline{/\theta_1 + \theta_2}$

thus, $z_7^2 = z_7 z_7 = r_7^2 \underline{/2\theta_7}$

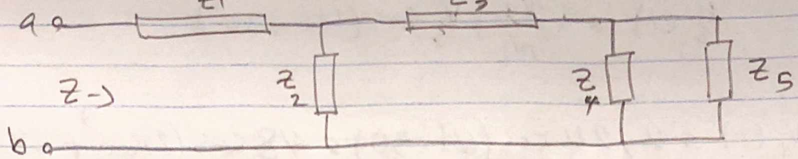
therfore,

$r_7^2 = 1.41 \Rightarrow r_7 = \sqrt{1.41} = 1.19$

$2\theta_7 = 135° \Rightarrow \theta_7 = 67.5°$

$\Rightarrow z_7 = (-1+j)^{1/2} = 1.19 \underline{/67.5°}$

7.36) Transferring to phasor domain:



$$Z_1 = 5\ \Omega, \quad Z_2 = \frac{1}{j\omega C} = -\frac{j}{400\times 2\times 10^{-3}} = -j1.25\ \Omega$$

$$Z_3 = j\omega L = j400\times 3\times 10^{-3} = j1.2\ \Omega$$

$$Z_4 = 5\ \Omega, \quad Z_5 = j\omega L = j400\times 9\times 10^{-3} = j3.6\ \Omega$$

$$Z_{45} = Z_4 \| Z_5 = \frac{j18}{5+j3.6} = 2.92\, e^{j54.24^\circ}\ \Omega$$

$$Z_{345} = Z_{45} + Z_3 = 2.92\, e^{j54.24^\circ} + j1.2 = 1.71 + j3.57\ \Omega$$

$$Z = Z_1 + Z_2 \| Z_{345} = 5 + (-j1.25) \| (1.71 + j3.57)$$

$$= 5 + \frac{(-j1.25)\times(1.71+j3.57)}{1.71+j2.32} = 5.32 - j1.69\ \Omega$$

$$\Rightarrow Z = 5.32 - j1.69\ \Omega$$

7.86) The voltage in secondary winding of the transformer is: $v_{s,1}(t) = \left(\frac{N_2}{N_1}\right) V_s \cos(\omega t + 30^\circ)$

$\Rightarrow v_{s,1}(t) = 2 \times 24 \cos(\omega t + 30) = 48 \cos(2\pi \times 10^3 t + 30^\circ)$

Thus, $V_{s,1} = 48$ is greater that zener voltage $V_Z = 42$ (V)

This will enable the zener diod to limit the output voltage at, $\boxed{v_{out} = V_Z = 42 \text{ (V)}}$

we know that, $R_Z \| R_L = \frac{20 \times 20^k}{20 + 20^k} = 19.98\ \Omega$

From equation 7.147 in textbook we can find the peak-to-peak ripple voltage:

$$V_r = \frac{\left[(V_{s,1} - 1.4) - V_Z\right] T_{rect}}{R_s C} \times \frac{R_Z \| R_L}{R_s + R_Z \| R_L}$$

As $\omega = 2\pi \times 10^3 \Rightarrow \omega_{rect} = 2\omega = 4\pi \times 10^3$

$$\Rightarrow T_{rect} = \frac{2\pi}{\omega_{rect}} = \frac{2\pi}{4\pi \times 10^3} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3} = 0.5 \text{ ms}$$

$$\Rightarrow V_r = \frac{\left[(48 - 1.4) - 42\right]}{50 \times 0.1 \times 10^{-3}} \times (0.5 \times 10^{-3}) \times \frac{19.98}{50 + 19.98}$$

$\Rightarrow \boxed{V_r = 0.1313 \text{ (V)}}$ (peak-to-peak)