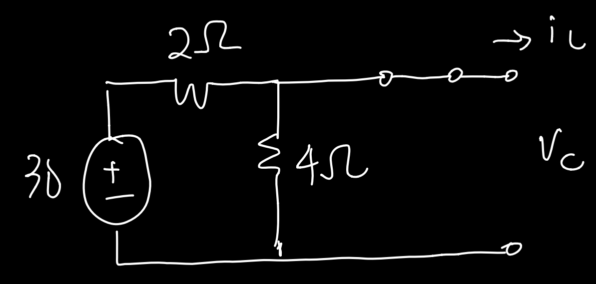
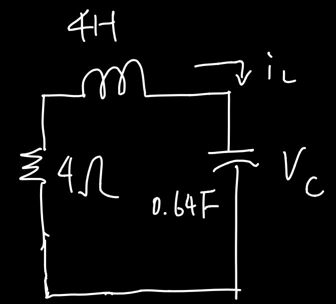
6.18)

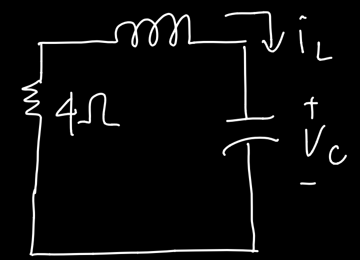
$t=0^-$



$t=0$



$t=\infty$



At the begining of the stage, the circuit is at steady state, and the inductor behaves like a short circuit, the capacitor behaves like a open circuit. As a result,

$$i_L(0^-) = 0 \text{ A}$$

$$V_C(0^-) = 30 \times \frac{4}{2+4} = 20 \text{ V}$$

At the time $t=0$, the switch change from close to open, and the current accorss inductor and

Voltage accross capacitor remains the same because they cannot change instantaneously.

$$i_L(0) = i_L(0^-) = i_C(0) = 0\,A$$

$$V_C(0) = V_C(0^-) = 20\ V$$

Notice this is a series RLC circuit and the capacitor will discharge all the way to 0 v.

$$V_C(\infty) = 0$$

With this in mind, the damping coeficient and resonant frequency is

$$\alpha = \frac{R}{2L} = \frac{4}{2 \cdot 4} = 0.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 0.64}} = 0.625$$

Since $\boxed{\alpha < \omega}$, it is underdamped.

$$\begin{cases} i_c(t) = C\dfrac{dV_c(t)}{dt} \\ V_c = e^{-\alpha t}\left[D_1 \cos\omega_d t + D_2 \sin\omega_d t\right] + V_c(\infty) \end{cases}$$

where $\omega_d^2 = \omega_0^2 - \alpha^2 = 0.140625 \Rightarrow \omega_d = 0.375 = \frac{3}{8}$

$$D_1 = V_C(0) - V_C(\infty) = 20 - 0 = 20\ V$$

$$D_2 = \frac{\frac{1}{C} i_C(0) + \alpha\left[V_C(0) - V_C(\infty)\right]}{\omega_d}$$

$$= \frac{\frac{1}{0.64} \times 0 + 0.5[20-0]}{\frac{3}{8}}$$

$$= \frac{80}{3}$$

$$\therefore \quad V_c = e^{-0.5t}\left[20\cos\frac{3}{8}t + \frac{80}{3}\sin\frac{3}{8}t\right]$$

$$\dot{V}_c = \cancel{-0.5e^{-0.5t} \cdot 20\cos\frac{3}{8}t} - e^{-0.5t} \cdot 20 \cdot \frac{3}{8} \cdot \sin\frac{3}{8}t$$

$$-0.5e^{-0.5t} \cdot \frac{80}{3} \cdot \sin\frac{3}{8}t + \cancel{e^{-0.5t} \cdot \frac{80}{3} \cdot \frac{3}{8} \cdot \cos\frac{3}{8}t}$$

$$= -\frac{500}{24} e^{-0.5t} \sin\frac{3}{8}t$$

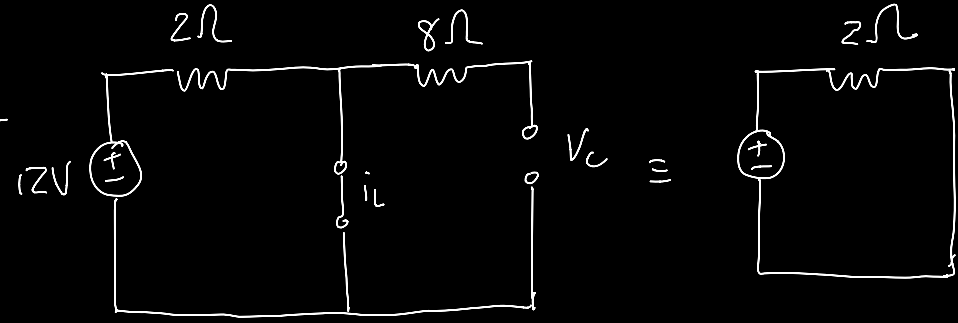
$$i_c = C\dot{V}_c$$

$$= 0.64\left(-\frac{500}{24} e^{-0.5t} \sin\frac{3}{8}t\right)$$

$$= -\frac{40}{3} e^{-0.5t} \sin\left(\frac{3}{8}t\right)$$

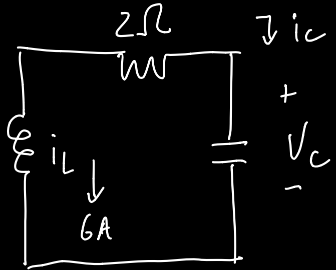
6.22)

$t=0^-$



$$i_L(0^-) = \frac{12V}{2} = 6A, \quad V_c(0^-) = 0$$

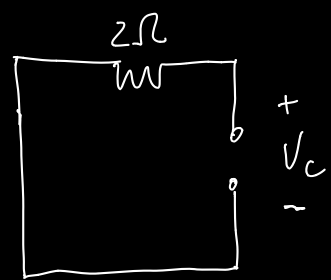
$t=0$,



$$V_c(0) = V_c(0^-) = 0$$

$$i_L(0) = i_L(0^-) = i_c(0) = 6A$$

$t=\infty$,



$$V_c(\infty) = 0\,V$$

$$i_L(\infty) = 0\,A$$

Remember that inductor and capacitor store energy, at $t=0$, the energy is stored in the inductor and the capacitor is fully discharged. At $t=\infty$, inductor fully discharged and capacitor was charged at begining and discharged in the end because all energy is dissipated by the resistor.

Let's calculate α (damping coeficient) and ω (renonant frequency) first to find out what type of damping is this RLC circuit. $\alpha = \frac{R}{2L} = \frac{8}{2\cdot 2} = 2$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\cdot 0.1}} = 2.23$

Since $\alpha < \omega$, the circuit is underdamped. We apply rule according to Table 6.1 (like what we did in problem 6.18)

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 1$$

$$D_1 = V_c(0) - V_c(\infty) = 0 + 12 = 0V$$

$$D_2 = \frac{\frac{1}{C} i_c(0) + \alpha\,[V_c(0) - V_c(\infty)]}{\omega_d}$$

$$= \frac{-1}{0.1} \cdot 6 + 2 \cdot (0)$$

$$= -60$$

$\therefore$ $$V_c(t) = e^{-\alpha t}\,[D_1 \cos \omega_d t + D_2 \sin \omega_d t] + V_c(\infty)$$

$$= e^{-2t} \cdot (-60) \cdot \sin t = -60 e^{-2t} \sin(t)$$

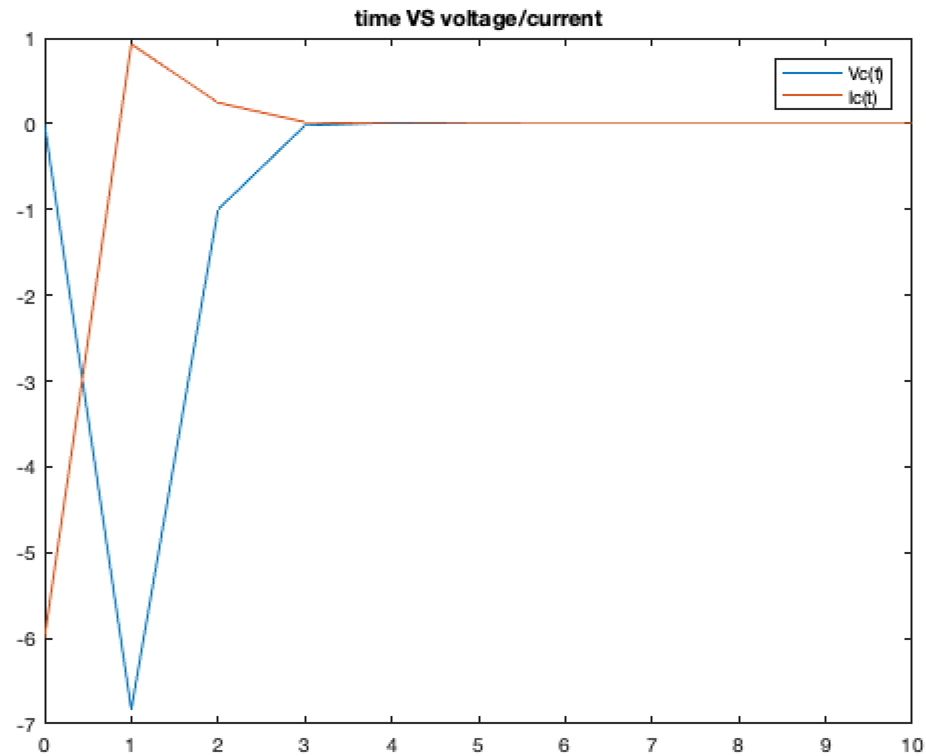
$$\dot{V}_c(t) = -2e^{-2t}(-60) \sin t + e^{-2t} \cdot (-60) \cdot \cos t$$

$$i_c(t) = C\,\dot{V}_c(t) = 0.1\,(120\, e^{-2t} \sin t - 60 e^{-2t} \cos t)$$

$$= 12 e^{-2t} \sin t - 6 e^{-2t} \cos t$$

$$= (12 \sin t - 6 \cos t)\, e^{-2t}$$

```
close all; clear all;
t = 0:10;
vc = -60*exp(-2*t).*sin(t);
ic = (12*sin(t) - 6*cos(t)).*exp(-2*t); figure;
plot(t, vc); hold on;
plot(t,ic);
legend('Vc(t)','Ic(t)');
title('time VS voltage/current');
```



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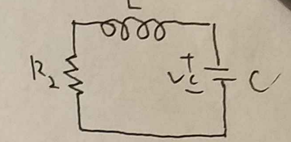
6.12

$t=0^-$ $\quad V_c(t=0^-) = V_0 \cdot \frac{R_2}{R_1+R_2} = 9\,V$

$I_L(t=0^-) = 0$

$t=0^+$ $\quad V_c(t=0^+) = V_c(t=0^-) = 9\,V$

$I_c(t=0^+) = I_L(t=0^+) = I_L(t=0^-) = 0$



$t=\infty$ $\quad V_c(t=\infty) = 0$

$I_c(t=\infty) = 0$

when $t>0$ $\quad \begin{cases} R_2 i_c + L\frac{di_c}{dt} + V_c = 0 \\ i_c = C\frac{dV_c}{dt} \end{cases} \Rightarrow R_2 C \frac{dV_c}{dt} + LC\frac{d^2V_c}{dt^2} + V_c = 0$ ①

Assume $V_c(t) = Ae^{st}$, ① → ②: $R_2CAse^{st} + LCAs^2e^{st} + Ae^{st} = 0$

$\Rightarrow V_c(t) = A_1e^{s_1t} + A_2e^{s_2t}$

$s_1 = \frac{-R_2C + \sqrt{(R_2C)^2 - 4\times LC}}{2\times LC} = -6+\sqrt{11} \approx -2.68$

$s_2 = \frac{-R_2C - \sqrt{(R_2C)^2 - 4\times LC}}{2\times LC} = -6-\sqrt{11} \approx -9.32$

when $t=0^+$ $\quad \begin{cases} V_c(0^+) = A_1 + A_2 = 9 \\ I_c(0^+) = C\frac{dV_c}{dt}\Big|_{t=0^+} = C(S_1A_1+S_2A_2) = 0 \end{cases}$

$\Rightarrow \begin{cases} A_1 = \frac{99+54\sqrt{11}}{22} \approx 12.64 \\ A_2 = \frac{99-54\sqrt{11}}{22} \approx -3.64 \end{cases}$

$\therefore V_c(t) \approx 12.64\,e^{-2.68t} - 3.64\,e^{-9.32t}$

6.16

$t=0^-$ $V_C(0^-)=V_0$

switch ~~before~~ at 1 $I_L(0^-)=0$

$t=0^+$ $V_C(0^+)=V_C(0^-)=V_0$

Switch at 2 $I_L(0^+)=I_L(0^-)=0$

$I_C(0^+)=I_L(0^+)$

$$\begin{cases} L\dfrac{di_C}{dt}+V_C=0 \\ i_C=C\dfrac{dV_C}{dt} \end{cases} \Rightarrow LC\frac{d^2V_C}{dt}+V_C=0$$

$$\Rightarrow V_C(t)=A\cos(\omega t)+B\sin(\omega t),\quad \omega=\frac{1}{\sqrt{LC}}$$

$t=0^+$, $V_C(0^+)=A=V_0 \Rightarrow A=V_0$

$$I_C(0^+)=C\frac{dV_C}{dt}\Big|_{t=0^+}=BC=0 \Rightarrow B=0$$

$$\therefore V_C(t)=V_0\cos(\omega t),\quad \omega=\frac{1}{\sqrt{LC}}$$