solution

EE101 Final Examination, March 20, 2019

Name________________________ Student ID_________________________

2 page of formulas and tables only are allowed. But, you must show all details of your work even when you apply formulas to get full credits. Otherwise, you will lose points.

Problem 1 [10] ____________

Problem 2 [10] ____________

Problem 3 [10] ____________

Problem 4 [10] ____________

Problem 5 [10] ____________

Problem 6 [10] ____________

Problem 7 [10] ____________

Problem 8 [10] ____________

Problem 9 [10] ____________

Problem 10[10] ____________

Bonus Q1 [5] ____________

Bonus Q2 [5] ____________

____

TOTAL [100] ____________

Bonus [10] ____________

Screening Question:

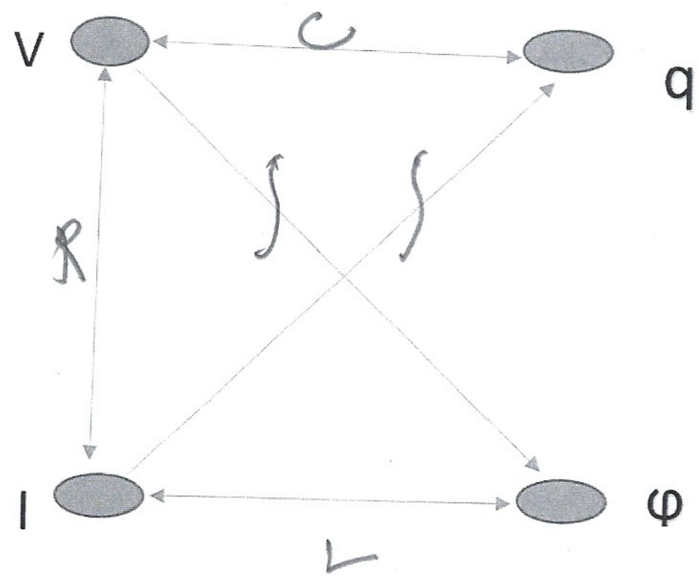
Write down the voltage-current relations (voltage expressed in terms of current) for

Resistor R: $v = Ri$

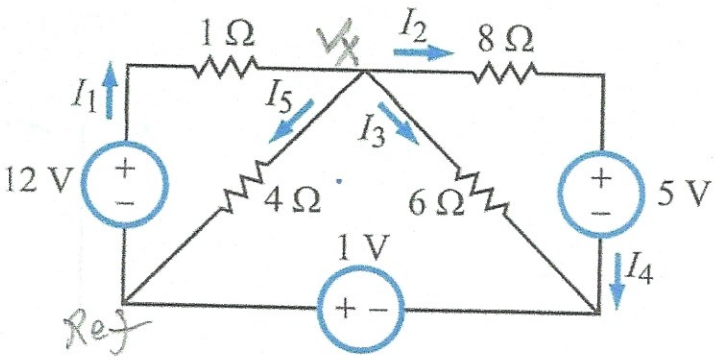
Inductor L: $v = L\frac{di}{dt}$

Capacitor C: $i = C\frac{dv}{dt}$

Insert R, L, C into 3 links below, and integration or differentiation signs.



[1] (10 points) Consider the following circuit with three independent voltage sources. Find the power consumed in the 4Ω resistor.



Answer= 592 [W].

$$\frac{12-V_x}{1} = \frac{V_x}{4} + \frac{V_x-(-1)}{6} + \frac{V_x-(5-1)}{8}$$

$$\times 24 \Rightarrow 24(12-V_x) = 6V_x + 4(V_x+1) + 3(V_x-4)$$

$$24\times 12 + 2\times 4 = (24+6+4+3)V_x$$

$$V_x = \frac{296}{37} = 8$$

$$P_{4\Omega} = \frac{V_x^2}{4} = \frac{8^2}{4} = 16\,[W]$$

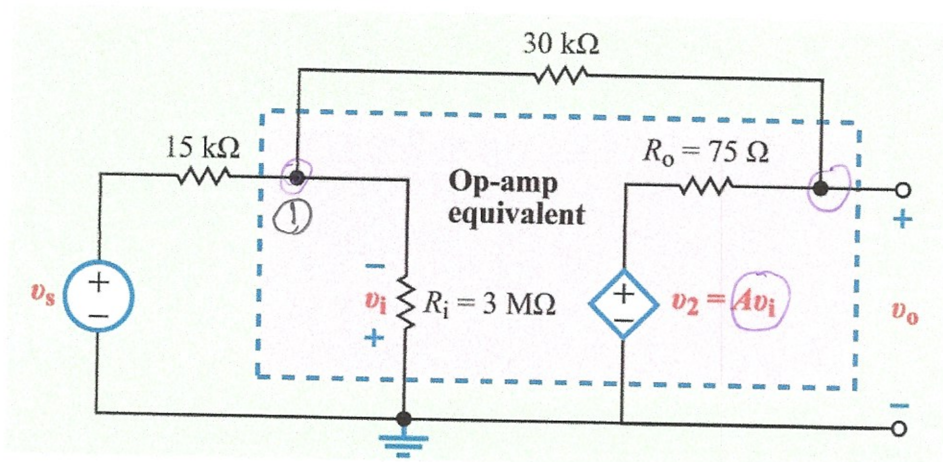
ans

not correct answer -5

correct answer but not correct step 8

correct answer & steps 10

[2] (10 points) Find the voltage gain G = v_o/v_s for A=1001.



1 — At node ①, KCL: $\dfrac{v_s-(-v_i)}{15K} = \dfrac{-v_i - v_o}{30K} + \dfrac{-v_i}{3000K}$

$\times 3000K \Rightarrow \quad 200(v_s + v_i) = -100(v_i + v_o) - v_i \quad (1)$

2 — Also $v_o = Av_i + 75\left(\dfrac{-v_i - v_o}{30K}\right)$

$v_o = 1001 v_i - \dfrac{v_i + v_o}{400}$

$= \left(1001 - \dfrac{1}{400}\right)v_i - \dfrac{v_o}{400}$

$v_o\left(1 + \dfrac{1}{400}\right) = \left(1001 - \dfrac{1}{400}\right)v_i$

$\times 400 \Rightarrow \quad 401 v_o = (1001 \times 400 - 1)v_i$

$v_i = \dfrac{401}{1001 \times 400 - 1} v_o \quad (2)$

From (1) & (2),

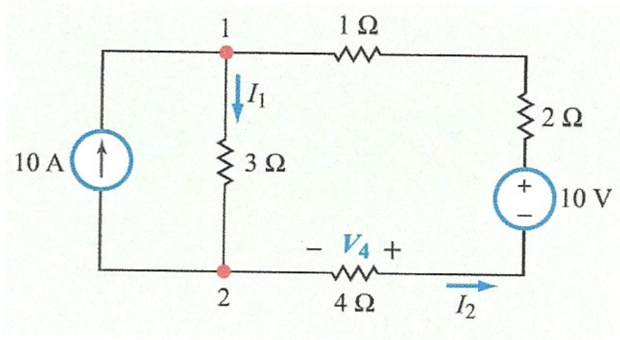
$200 v_s = -301 v_i - 100 v_o$

$= -\left[301\left(\dfrac{401}{1001 \times 400 - 1}\right) + 100\right] v_o$

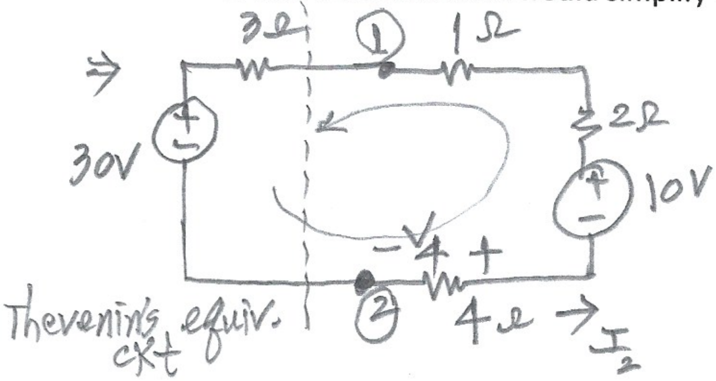
$\dfrac{v_o}{v_s} = \dfrac{200}{-\left[301\left(\dfrac{401}{1001\times 400 - 1}\right) + 100\right]} = -\dfrac{200}{100.3} = -1.99 = G$

ans

[3] (10 points) Find the current I_2 in the circuit shown below.



Hint: A source transformation would simplify the problem.



KVL

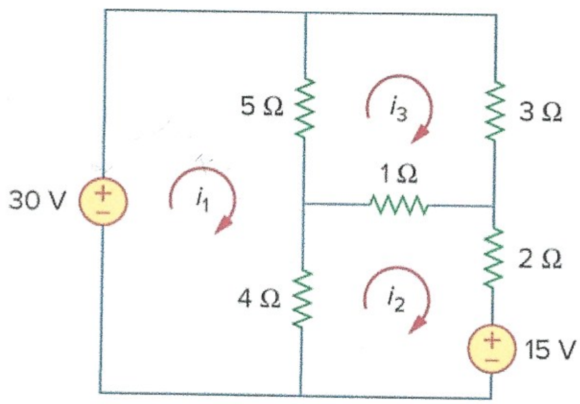
$$4I_2 - 10 + 2I_2 + 1I_2 + 3I_2 + 30 = 0$$

$$10I_2 = -20$$

$$I_2 = -2\,[A]$$

ans

[4] (10 points) Find the power supplied by the 30V voltage source by finding mesh currents i_1, i_2 and i_3.



$P_{30V} = 30(-i_1)$

(a) (5 points) Write down a matrix equation to find mesh currents as Ax=b, where A is a 3X3 matrix.

$$-30 + 5(i_1 - i_3) + 4(i_1 - i_2) = 0$$
$$4(i_2 - i_1) + 1(i_2 - i_3) + 2i_2 + 15 = 0$$
$$5(i_3 - i_1) + 3i_3 + 1(i_3 - i_2) = 0$$

$$\underbrace{\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{\tilde{x}} = \underbrace{\begin{bmatrix} 30 \\ -15 \\ 0 \end{bmatrix}}_{\tilde{b}} \qquad \tilde{A}\,\tilde{x} = \tilde{b}$$

(b) (5 points) Then find the mesh current i_1 and then the power supplied by the 30V voltage source.

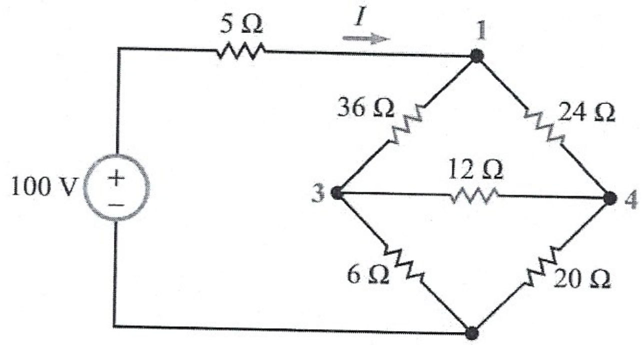
$$i_1 = \frac{\begin{vmatrix} 30 & -4 & -5 \\ -15 & 7 & -1 \\ 0 & -1 & 9 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{vmatrix}} = \frac{\begin{cases} 30\times7\times9 + (-1)(-15)(-5) + 0 \\ -0 - (-15)(-4)9 - 30(-1)(-1) \\ = 1245 \end{cases}}{\begin{cases} 9\times7\times9 + (-4)(-1)(-5) + (-5)(-4)(-1) \\ -(-5)(7)(-5) - (-4)(-4)9 - 9(-1)(-1) \\ = 567 - 20 - 20 - 175 - 144 - 9 \\ = 199 \end{cases}}$$

$= 6.256$ [A]

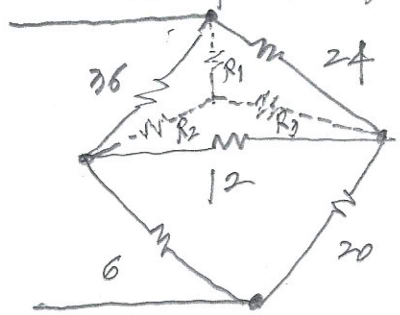
$P_{30V} = -30(6.256) = -187.69$ [W]

ans 187.69 [W] is supplied

[5] (10 points) Find the current I in the circuit below.



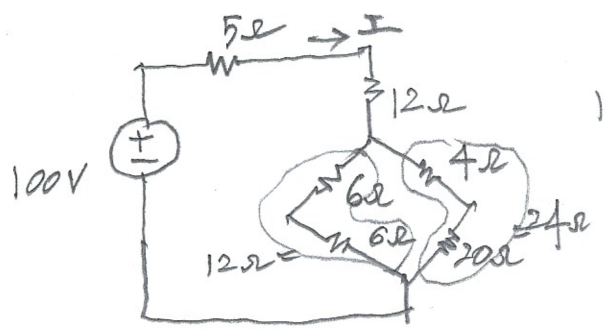
$\Delta - Y$ Transformation

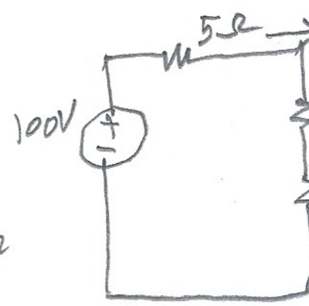


$$R_1 = \frac{36 \times 24}{36+24+12} = \frac{36 \times 24}{72} = 12\,\Omega$$

$$R_2 = \frac{36 \times 12}{36+24+12} = \frac{36 \times 12}{72} = 6\,\Omega$$

$$R_3 = \frac{24 \times 12}{36+24+12} = \frac{24 \times 12}{72} = 4\,\Omega$$





$$12 \| 24 = \frac{12 \times 24}{12+24} = 8\,\Omega$$

$$I = \frac{100\text{V}}{(5+12+8)\,\Omega} = \frac{100\text{V}}{25\,\Omega} = 4\,[\text{A}]$$

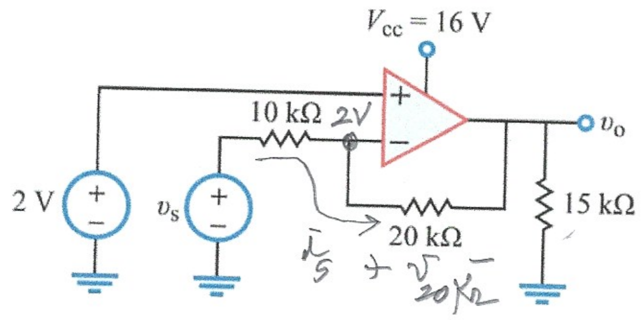
ans

[6] (10 points) Determine the range of the source voltage v_s for the output voltage v_o to be in the linear range without saturation. Assume that the Op amp is ideal.

Correct: 10

incorrect V_S : 5

try: 3



$$i_s = \frac{v_s - 2}{10K\Omega}$$

$$v_{20K\Omega} = 20K\Omega\, i_s = 20K\Omega\left(\frac{v_s - 2}{10K\Omega}\right) = 2v_s - 4$$

$$v_o = -v_{20K\Omega} + 2 = -2v_s + 4 + 2 = -2v_s + 6$$

$$-16V \le v_o \le 16V$$

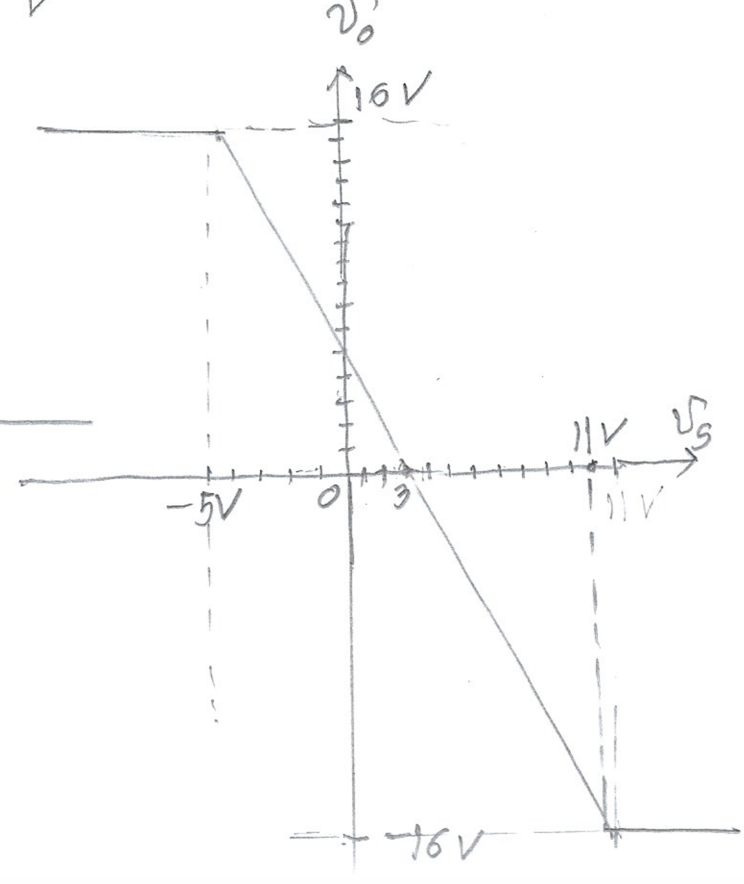
$$-16 \le -2v_s + 6 \le 16V$$

$$-22 \le -2v_s \le 10V$$

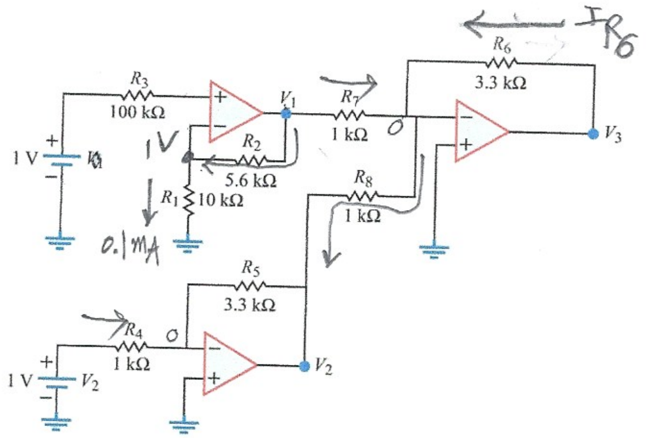
$$22 \ge 2v_s \ge -10V$$

$$11V \ge v_s \ge -5V$$

ans



[7] (10 points) Find the output voltage v_3 in the ideal Op amp circuit below.



correct : 10 / 8

incorrect : 5

try : 3

other : 0

$$V_1 = 1 + 5.6K(0.1\,mA) = 1.56\,V$$

$$I_{R_7} = \frac{V_1 - 0}{1\,K\Omega} = 1.56\,mA$$

$$I_{R_4} = \frac{1-0}{1\,K\Omega} = 1\,mA = I_{R_5}$$

$$V_2 = -3.3 \times I_{R_5} = -3.3\,V$$

$$I_{R_8} = \frac{0 - V_2}{1\,K\Omega} = \frac{3.3\,V}{1\,K\Omega} = 3.3\,mA$$

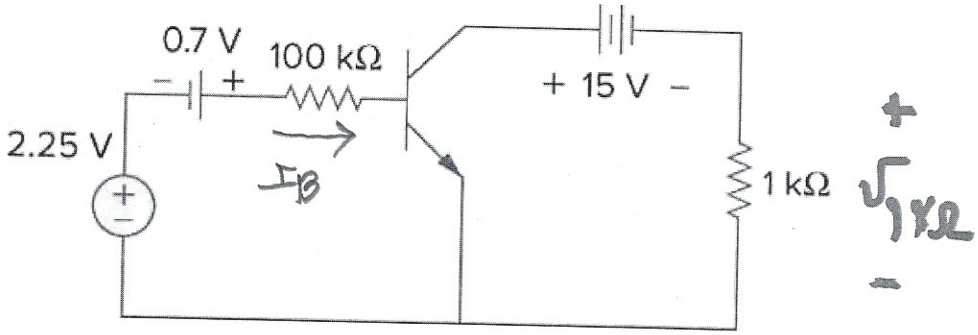
$$I_{R_6} = I_{R_8} - I_{R_7} = 3.3\,mA - 1.56\,mA = 1.74\,mA$$

$$V_3 = I_{R_6} R_6 = 1.74\,mA\,(3.3\,K\Omega) = +\underline{5.742\,V}$$

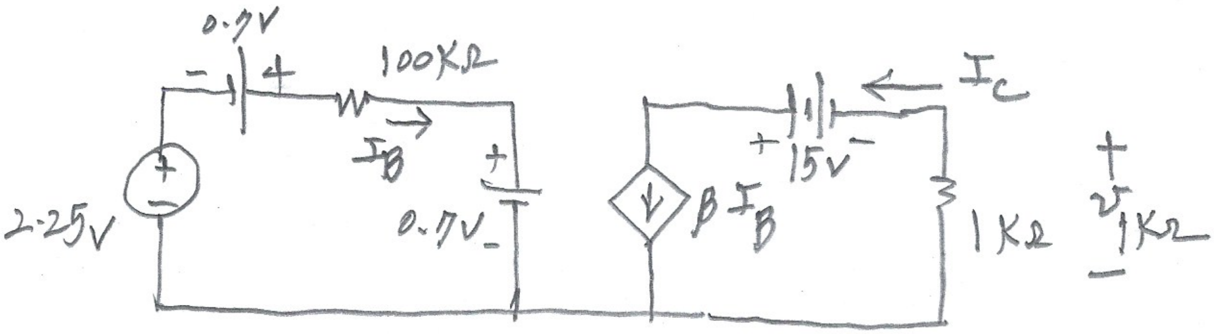
ans

[8] (10 points) Find the voltage across the load resistor 1KΩ and the power delivered to it.

For BJT, $V_{BE(on)} = 0.7\,V$, $\beta = 100$



Answer $V_{1K\Omega}$ = $-2.25\,[V]$ Power = $5.06\,[mW]$



Using Mesh analysis, $-2.25 - 0.7 + 100K\Omega\, I_B + 0.7 = 0$

$$\Rightarrow I_B = \frac{2.25\,V}{100K\,\Omega} = 22.5 \times 10^{-6}\,[A] = 22.5\,[\mu A]$$

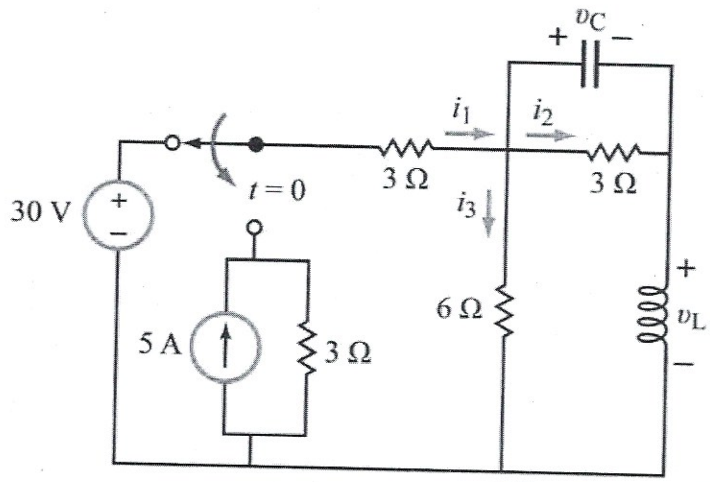
$$I_C = \beta I_B = 100 \times 22.5 \times 10^{-6} = 2.25\,[mA]$$

$$V_{1K\Omega} = -1K\Omega\, I_C = -1\,K\Omega \times 2.25\,[mA] = \underline{-2.25\,[V]}$$

$$P_{1K\Omega} = I^2 R = \frac{V_{1K\Omega}^2}{1K\Omega} = \frac{2.25^2}{1 \times 10^3} = \underline{5.06\,[mW]}$$

[9] (10 points) Find $v_C(t)$ for $t > 0$ in the circuit below. ~~Choose the time-domain solution method using the table attached.~~ Find a second-order differential equation in terms of $v_C(t)$.

$C = 1F, \; L = 1H$



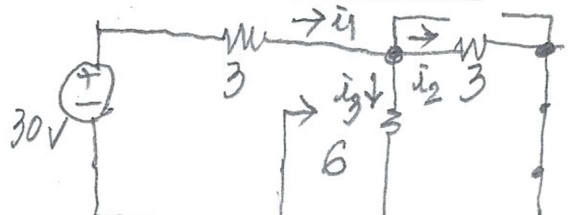
ans

$$\frac{d^2v_C}{dt^2} + 3\frac{1}{3}\frac{dv_C}{dt} + 2v_C = 7.5$$

$v_C(0) = 12$ [V]

$v_C'(0) = 0$

For $t < 0$



$R_{eq} = 2\ \Omega$

$i_1(0^-) = \frac{30[V]}{(3+2)[\Omega]} = 6$ [A]

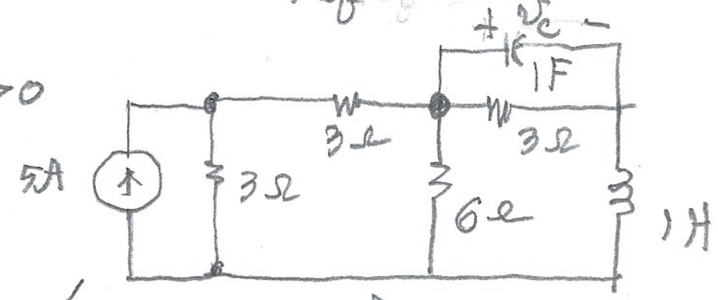
$i_2(0^-) = i_L(0^-) = 6 \times \frac{6}{6+3}$

$= 4$ [A] $= i_L(0)$

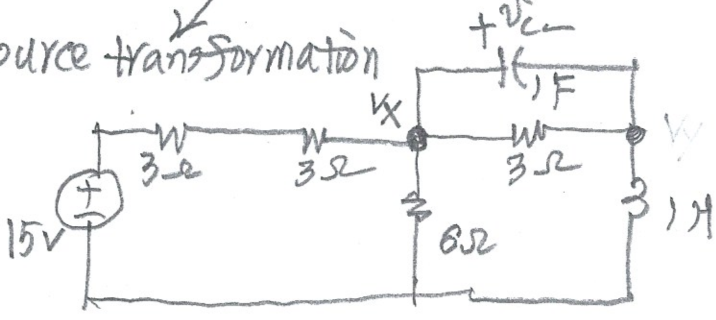
$v_C(0^-) = 3 \times 4 = 12$ [V]

$= v_C(0)$

For $t > 0$



source transformation



KCL at V_x node

$$\frac{15 - V_x}{6} = \frac{V_x}{6} + 1\frac{dv_C}{dt} + \frac{v_C}{3} \quad (1)$$

$$V_x = V_C + 1\frac{di_L}{dt}$$

$$= V_C + \frac{d}{dt}\left(\frac{dv_C}{dt} + \frac{v_C}{3}\right)$$

$$= V_C + \frac{d^2v_C}{dt^2} + \frac{1}{3}\frac{dv_C}{dt} \quad (2)$$

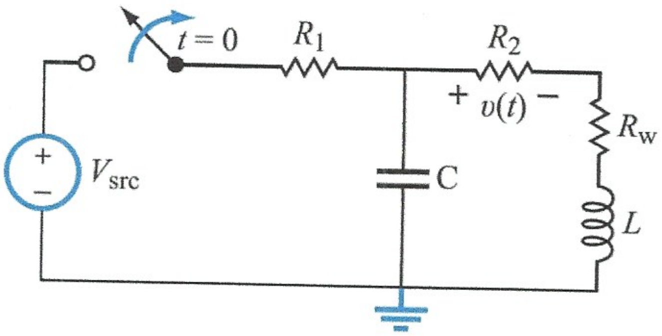
Multiply both sides of (1) by 6 ⇒ $15 - V_x = V_x + 6\frac{dv_C}{dt} + 2v_C \quad (1)'$

From (1)' and (2) ⇒ $15 = 2\left(V_C + \frac{d^2v_C}{dt^2} + \frac{1}{3}\frac{dv_C}{dt}\right) + 6\frac{dv_C}{dt} + 2v_C$

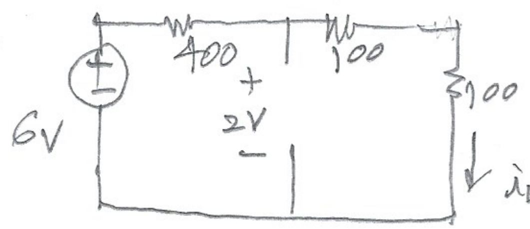
or $7.5 = \frac{d^2v_C}{dt^2} + 3\frac{1}{3}\frac{dv_C}{dt} + 2v_C$ ans

This problem was supposedly right problem for [9].

[9] (10 points) Find v_C(t) for t > 0 in the circuit below. Choose the time-domain solution method using the table attached. Use $V_{src} = 6\ V,\ R_{1,2,w} = 400, 100, 100\ \Omega$, C= 1 μF, L= 10 mH.



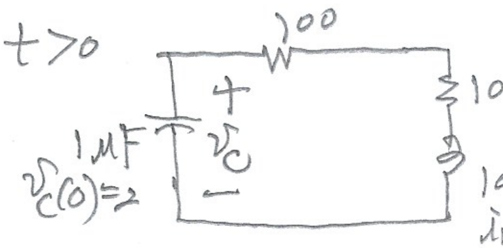
At t=0-,



$i_L(0^-) = \frac{6}{400+100+100} = \frac{6}{600} = 10\ mA$

$v_C(0^-) = 10\ mA \times 200\ \Omega = 2\ V$

At t>0



$v_C = i_L(100+100) + 10\times10^{-3}\frac{d}{dt}i_L \quad (1)$ where $i_L = -i_C$

$i_L = -C\frac{dv_C}{dt} = -10^{-6}\frac{dv_C}{dt} \quad (2)$

$(2)\to(1) \Rightarrow v_C = 200\left(-10^{-6}\frac{dv_C}{dt}\right) + 10^{-2}\frac{d}{dt}\left(-10^{-6}\frac{dv_C}{dt}\right)$

$\times(-10^{8}) \Rightarrow -10^{8}v_C = 2\times10^{4}\frac{dv_C}{dt} + \frac{d^2v_C}{dt^2}$

$\frac{d^2v_C}{dt^2} + 2\times10^{4}\frac{dv_C}{dt} + 10^{8}v_C = 0$

$\alpha = 10^4,\quad \omega_0 = \sqrt{10^8} = 10^4,\quad v_C(\infty) = 0$

$\alpha = \omega_0$ critically damped.

([9] solution continued)

$$v_C(t) = (B_1 + B_2 t)\, e^{-\alpha t} + \overset{0}{\overset{\parallel}{v_C(\infty)}}$$

$$B_1 = v_C(0) - v_C(\infty) = 2$$

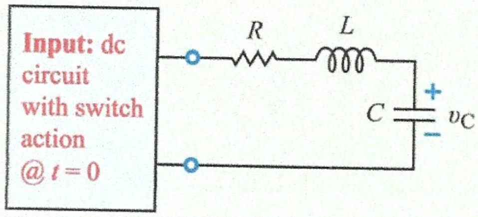
$$B_2 = \frac{1}{10^{-6}}\, i_C(0) + \alpha\left[v_C(0) - \overset{0}{\overset{\parallel}{v_C(\infty)}}\right]$$

$$= \frac{1}{10^{-6}}\left[-i_L(0) + \alpha\, v_C(0)\right] = \frac{1}{10^{-6}}\left[-10\times 10^{-3} + 10^{4}\times 2\right] = 2\times 10^{10}$$

$$v_C(t) = (2 + 2\times 10^{10} t)\, e^{-10^{4} t}$$

ans

Series RLC



Total Response

Overdamped ($\alpha > \omega_0$)

$$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + v_C(\infty)$$

$$A_1 = \frac{\frac{1}{C}\, i_C(0) - s_2[v_C(0) - v_C(\infty)]}{s_1 - s_2}$$

$$A_2 = \left[\frac{\frac{1}{C}\, i_C(0) - s_1[v_C(0) - v_C(\infty)]}{s_2 - s_1}\right]$$

Critically Damped ($\alpha = \omega_0$)

$$v_C(t) = (B_1 + B_2 t)e^{-\alpha t} + v_C(\infty)$$

$$B_1 = v_C(0) - v_C(\infty)$$

$$B_2 = \frac{1}{C}\, i_C(0) + \alpha[v_C(0) - v_C(\infty)]$$

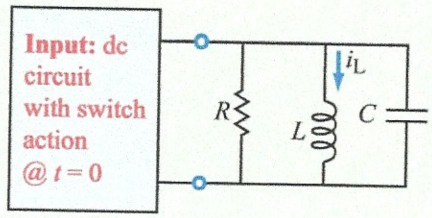
Underdamped ($\alpha < \omega_0$)

$$v_C(t) = e^{-\alpha t}(D_1 \cos \omega_d t + D_2 \sin \omega_d t) + v_C(\infty)$$

$$D_1 = v_C(0) - v_C(\infty)$$

$$D_2 = \frac{\frac{1}{C}\, i_C(0) + \alpha[v_C(0) - v_C(\infty)]}{\omega_d}$$

Parallel RLC



Total Response

Overdamped ($\alpha > \omega_0$)

$$i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)$$

$$A_1 = \frac{\frac{1}{L}\, v_L(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}$$

$$A_2 = \left[\frac{\frac{1}{L}\, v_L(0) - s_1[i_L(0) - i_L(\infty)]}{s_2 - s_1}\right]$$

Critically Damped ($\alpha = \omega_0$)

$$i_L(t) = (B_1 + B_2 t)e^{-\alpha t} + i_L(\infty)$$

$$B_1 = i_L(0) - i_L(\infty)$$

$$B_2 = \frac{1}{L}\, v_L(0) + \alpha[i_L(0) - i_L(\infty)]$$

Underdamped ($\alpha < \omega_0$)

$$i_L(t) = e^{-\alpha t}(D_1 \cos \omega_d t + D_2 \sin \omega_d t) + i_L(\infty)$$

$$D_1 = i_L(0) - i_L(\infty)$$

$$D_2 = \frac{\frac{1}{L}\, v_L(0) + \alpha[i_L(0) - i_L(\infty)]}{\omega_d}$$

Auxiliary Relations

$$\alpha = \begin{cases} \dfrac{R}{2L} & \text{Series RLC} \\[2ex] \dfrac{1}{2RC} & \text{Parallel RLC} \end{cases}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

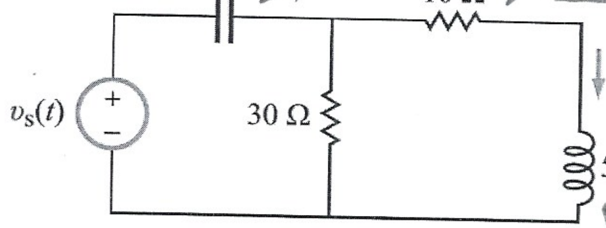
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

[10] (10 points) Find $i_a(t)$ for $v_S(t) = 20\sin(200t - 20^o)$ by using the phasor analysis method. Initial conditions are assumed to be zero.

$\frac{1}{j\omega C} = -j\frac{1}{200\times 200\times 10^{-6}} = \boxed{-j25}$

$\boxed{10}$



$j\omega L = j200(50\times 10^{-3}) = \boxed{j10}$

Hint:

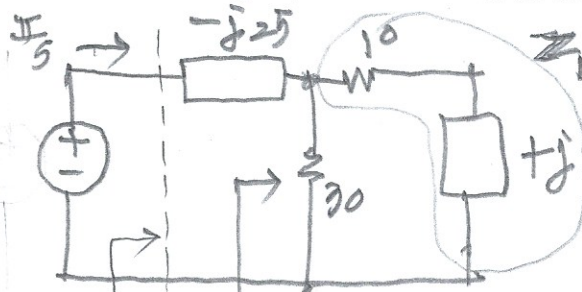
Step 1: Express v_S by using the phasor notation.

$v_S(t) = 20\sin(200t-20^\circ) = 20\cos(200t-20^\circ-90^\circ)$
$= 20\cos(200t-110^\circ)$
$\mathbb{V}_S = 20e^{-j110^\circ} = 20\angle -110^\circ$

Step 2: Find the current (in phasor form) flowing out of the voltage source.

Step 3: Apply the current division law to find i_a in phasor form. (see below)

Step 4: Convert i_a in phasor form into its time-domain expression. $= 0.674\cos(200t-58.15^\circ)$



$Z_S \quad Z_{eq} = 30 \| Z_1 = \frac{30(10+j10)}{30+(10+j10)} = \frac{30(10+j10)}{40+j10} = \frac{30(1+j1)}{4+j1}$

$Z_S = -j25 + Z_{eq} = -j25 + \frac{30(1+j1)}{4+j1}$

$= \frac{-j25(4+j1)+30+j30}{4+j1} = \frac{55-j70}{4+j1}$

$\mathbb{I}_S = \frac{\mathbb{V}_S}{Z_S} = \frac{20e^{-j110^\circ}(4+j1)}{55-j70}$

$\mathbb{I}_a = \mathbb{I}_S \frac{30}{Z_1+30} = \left[\frac{20e^{-j110^\circ}(\cancel{4+j1})}{55-j70}\right]\frac{\overset{3}{\cancel{30}}}{\underbrace{(10+j10)+30}_{=\cancel{10}(\cancel{4+j1})}}$

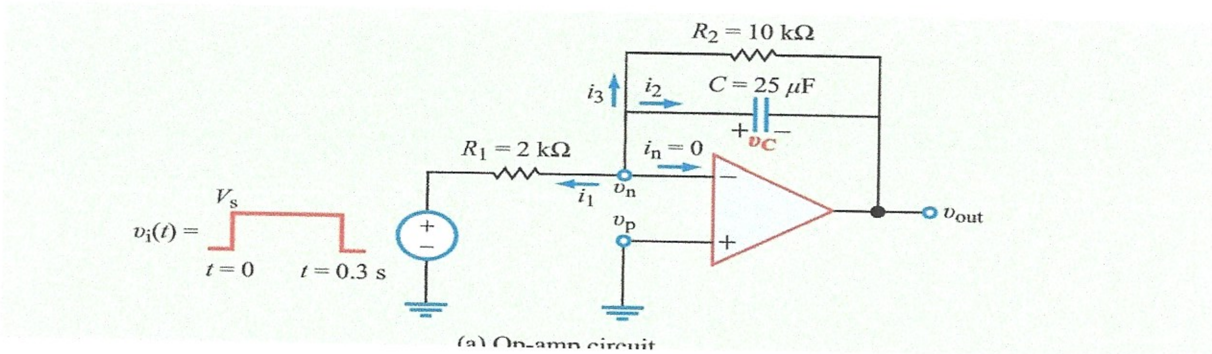
$= \frac{20e^{-j110^\circ}}{5(11-j14)} \times 3 = 12\frac{e^{-j110^\circ}}{\sqrt{11^2+(-14)^2}\, e^{j\tan^{-1}\left(\frac{-14}{11}\right)}}$

$= \frac{12}{17.80}e^{-j110^\circ-(-j51.85^\circ)} = 0.674e^{-j58.15^\circ}$

$\Rightarrow i_a(t) = 0.674\cos(200t-58.15^\circ)$

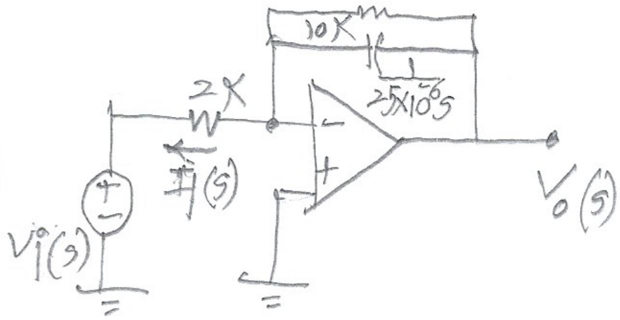
Bonus Q1 (5 points) Find v_{out} (t) for v_c (0) = 0 using Laplace transform method.

Hint: Laplace transform u (t) = 1/S and that of e^{-at} = 1/(S+a).



$$v_i(t) = V_s\left(u(t) - u(t-0.3)\right)$$

$$V_i(s) = V_s\left(\frac{1}{s} - \frac{1}{s}e^{-0.3s}\right)$$



$$I_1(s) = \frac{0 - V_i(s)}{2K}$$

$$I_2(s) = \frac{0 - V_o(s)}{1/25\times10^{-6}s} = -25\times10^{-6} s V_o(s)$$

$$I_3(s) = \frac{0 - V_o(s)}{10K}$$

$$KCL \Rightarrow I_1(s) + I_2(s) + I_3(s) = 0$$

$$-\frac{V_i(s)}{2K} - 25\times10^{-6} s V_o(s) - \frac{V_o(s)}{10K} = 0$$

$$V_o(s)\left[25\times10^{-6}s + \frac{1}{10K}\right] = -\frac{V_i(s)}{2K}$$

$$\times 10K \Rightarrow V_o(s)\left[25\times10^{-3}s + 1\right] = -5V_i(s)$$

$$V_o(s) = -\frac{5V_i(s)}{25\times10^{-3}s + 1}\ (\times 40) = -\frac{200V_i(s)}{s+40} = -\frac{200\left(\frac{1}{s} - \frac{1}{s}e^{-0.3s}\right)}{s+40}$$

$$= -200\left[\frac{1}{s(s+40)} - \frac{1}{s(s+40)}e^{-0.3s}\right] = -200\left[\frac{\frac{1}{40}}{s} + \frac{-\frac{1}{40}}{s+40} - e^{-0.3s}\left(\frac{\frac{1}{40}}{s} + \frac{-\frac{1}{40}}{s+40}\right)\right]$$

$$v_o(t) = -5\left(1 - e^{-40t}\right)u(t) + 5\left(1 - e^{-40(t-0.3)}\right)u(t-0.3)$$

Personalized

Bonus Q2 (5 points): List 5 most significant concepts you have learned in EE101 this quarter.

Concept 1:

Concept 2:

Concept 3:

Concept 4:

Concept 5: