EE 101 Lecture 7, Jan 28, 2019.

HW # 4

[1] prob. 3.58
[2] prob. 3.62
[3] prob. 3.64
[4] prob. 3.68
[5] prob. 3.72
[6] prob. 3.74
[7] prob. 3.77
[8] prob. 3.80

Prob. 2.46

Determine I

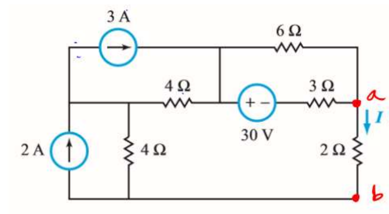


Figure P2.46: Circuit for Problem 2.46.

Let's solve this prob. by ~~first finding~~ the Norton's equivalent circuit for a-b port (terminals)

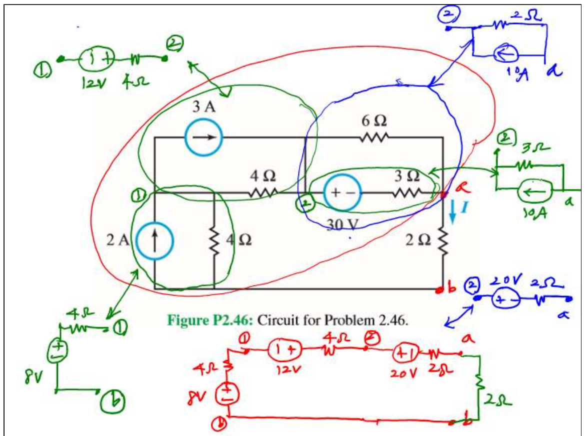
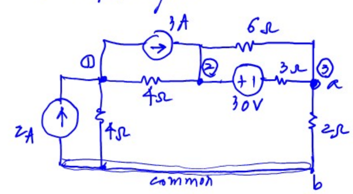


Figure P2.46: Circuit for Problem 2.46.

Nodal Analysis



KCL at ① $\quad 2 = 3 + \frac{V_1}{4} + \frac{V_1 - V_2}{4}$ (1)

KCL at ② $\quad 3 + \frac{V_1 - V_2}{4} = \frac{V_2 - V_3}{6} + \frac{V_2 - 30 - V_3}{3}$ (2)

KCL at ③ $\quad \frac{V_2 - V_3}{6} + \frac{V_2 - 30 - V_3}{3} = \frac{V_3}{2}$ (3)

$(1) \rightarrow \quad 2V_1 - V_2 = -4 \quad (1)'$

$(2) \rightarrow \quad -3V_1 + 9V_2 - 6V_3 = 156 \quad (2)'$

$(3) \rightarrow \quad V_2 - 2V_3 = 20 \quad (3)'$

$3 \times (1)' + 2 \times (2)' \rightarrow$

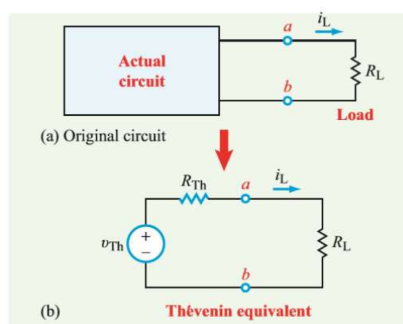
$$\begin{array}{r} 6V_1 - 3V_2 \qquad\qquad = -12 \\ -6V_1 + 18V_2 - 12V_3 = 312 \\ \hline 15V_2 - 12V_3 = 300 \quad (4) \end{array}$$

$(4) - 15 \times (3)' \rightarrow$

$$\begin{array}{r} 15V_2 - 12V_3 = 300 \\ -(15V_2 - 30V_3 = 300) \\ \hline 18V_3 = 0 \end{array} \quad \boxed{V_3 = 0\ V}$$

From $(3)'$ $\boxed{V_2 = 20\ V}$

From $(1)'$ $2V_1 - 20 = -4 \Rightarrow \boxed{V_1 = 8\ V}$



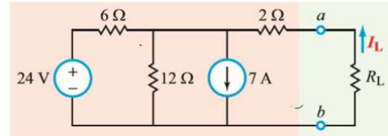
By analyzing the circuit configuration in Fig. 3-22(b) to find i_{sc} or, measuring i_{sc} with an ammeter, we can apply Eq. (3.33) to find R_{Th},

$$R_{Th} = \frac{v_{Th}}{i_{sc}}. \tag{3.34}$$

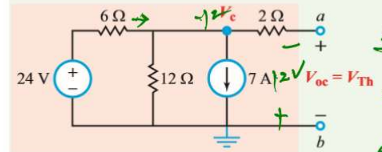
The only potential problem with this type of measurement is that when short-circuiting the source circuit, the current threshold of the ammeter may be exceeded (if the output resistance of the source circuit is very small).

This method is applicable to any circuit with at least one independent source, regardless of whether or not it contains dependent sources.

► The Thévenin voltage v_{Th} is obtained by removing the load R_L (replacing it with an open circuit), and then measuring or computing the open-circuit voltage at the same terminals. The short-circuit current i_{sc} is obtained by replacing the load with a short circuit and then measuring or computing the short-circuit current flowing through it (Fig. 3-22). ◄



(a) Original circuit



(b) Replacing R_L with open circuit

$$\frac{24 - V_c}{6} = 7 + \frac{V_c}{12}$$

$$\Downarrow$$

$$48 - 2V_c = 84 + V_c$$

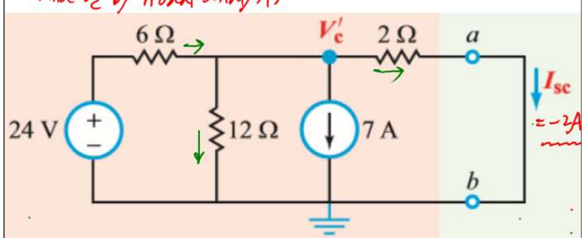
$$3V_c = -36$$

$$V_c = -12V$$

$I_{sc} = \frac{V_c'}{2\Omega}$

Find v_c' by nodal analysis

$I_{sc} = \frac{(-4)}{2} = -2A$



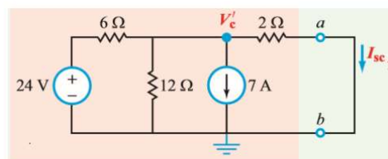
KCL at V_c' node:

$$\frac{24 - V_c'}{6} - \frac{V_c'}{12} - 7 = \frac{V_c'}{2}$$

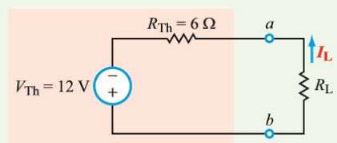
$(\times 12) \Rightarrow 48 - 2V_c' - V_c' - 7\times 12 = 6V_c'$

$9V_c' = -36$

$V_c' = -4[V]$



(c) Replacing R_L with short circuit

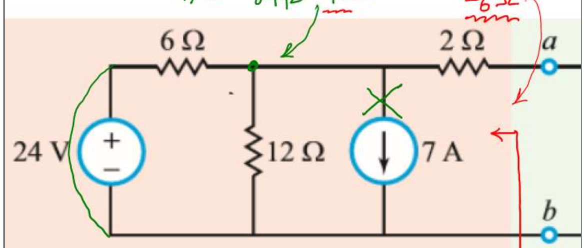


(d) Thévenin equivalent circuit

$$R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{-12V}{-2A} = 6\Omega$$

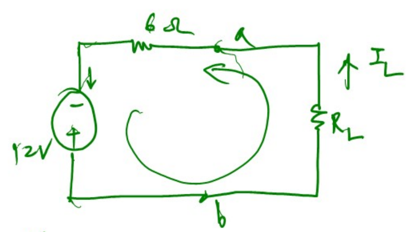
$6 // 12 = \frac{6\times 12}{6+12} = 4\Omega$

$4\Omega + 2\Omega = 6\Omega$



(a) Original circuit

set indep. sources to zero (Voltage source ← short, current source ← open)



KVL $-12 + R_L I + 6I = 0$

$I = \frac{12}{R_L + 6} = 1 \quad R_L = 6$

$0.8 \quad R_L = 9$

Thévenin and Norton Equivalency

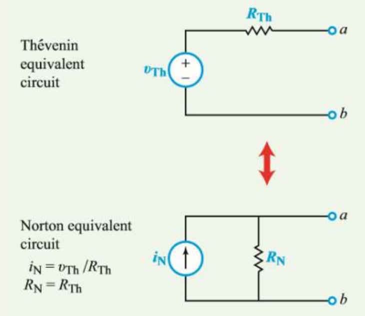


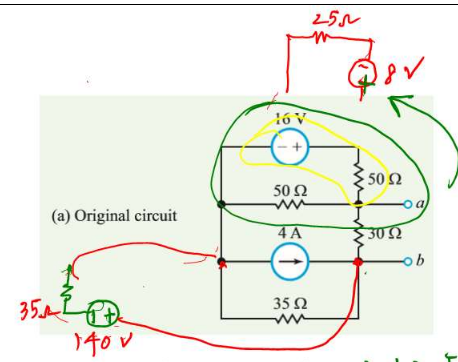
Figure 3-28: Equivalence between Thévenin and Norton equivalent circuits, consistent with the source transformation method of Section 2-3.4.

$V_{oc} = v_{TH}$

$I_{SC} = \frac{v_{Th}}{R_{TH}}$

$V_{oc} = R_N i_N = \cancel{R_{TH}} \frac{v_{TH}}{\cancel{R_{TH}}} = v_{TH}$

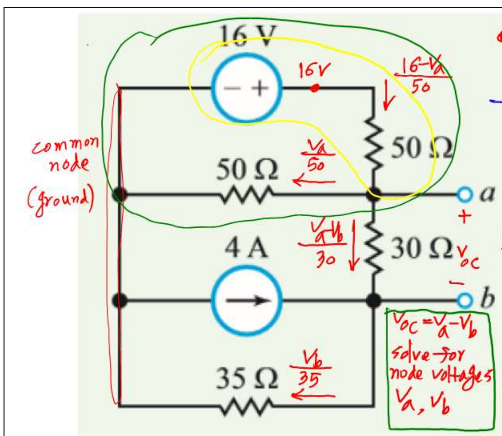
$i_{SC} = i_N = \frac{v_{TH}}{R_{TH}}$



$+8 + 25I + 35I - 140 + 30I = 0$

$90I = +132 \quad I = \frac{132}{90}$

$v = I \times 30 = 44\,V$

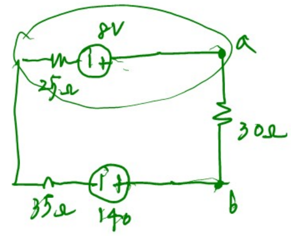


KCL at (a)

$\frac{16-V_a}{50} = \frac{V_a}{50} + \frac{V_a - V_b}{30}$ eq(1)

KCL at (b)

$\frac{V_a - V_b}{30} + 4 = \frac{V_b}{35}$ eq(2)



Eq (1) × 150 ⇒ $48 - 3V_a = 3V_a + 5V_a - 5V_b$

$48 = 11V_a - 5V_b \quad (1)'$

Eq (2) × 210 ⇒ $7V_a - 7V_b + 4 \times 210 = 6V_b$

$840 = -7V_a + 13V_b \quad (2)'$

$(1)' \times 13 + (2)' \times 5 \Rightarrow$

$48 \times 13 = 143V_a - 65V_b$

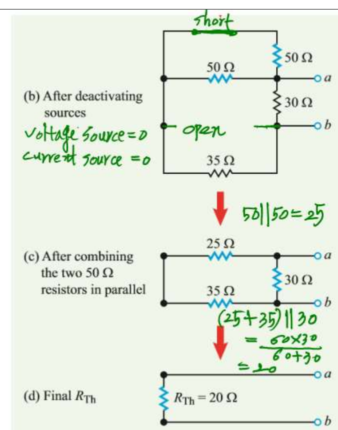
$840 \times 5 = -35V_a + 65V_b$

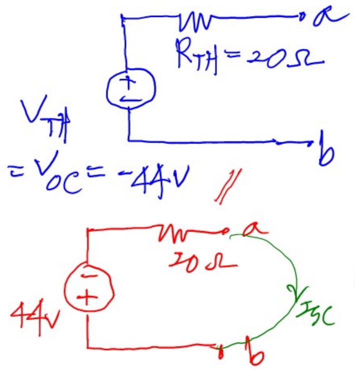
$4824 = 108V_a + 0$

$V_a = 4824/108 = \frac{134}{3} = 44\frac{2}{3}\,[V]$

From (1)', $V_b = \frac{11V_a - 48}{5} = \frac{11 \times \frac{134}{3} - 48}{5} = \frac{11 \times 134 - 48 \times 3}{3 \times 5} = \frac{1330}{3 \times 5} = \frac{266}{3} = 88\frac{2}{3}\,[V]$

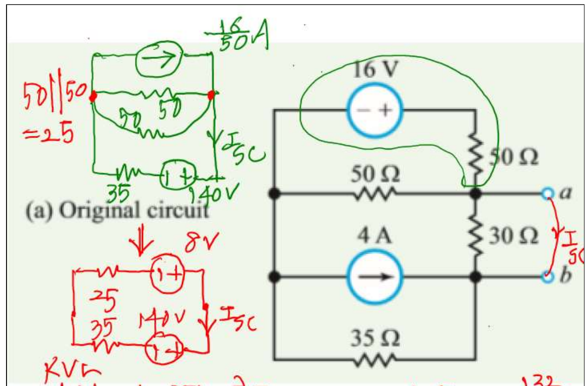
$V_{oc} = V_a - V_b = 44\frac{2}{3} - 88\frac{2}{3} = -44\,[V]$





$V_{TH} = V_{OC} = -44V$

$I_{SC} = -\frac{44}{20} = -2.2A$

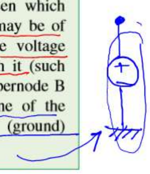


KVL
$+140 + (35+25) I_{SC} - 8 = 0 \Rightarrow I_{SC} = -\frac{132}{60} = -2.2A$

3-2.3 Supernodes

Occasionally, a circuit may contain a solitary voltage source nestled between two extraordinary nodes, with no other elements in series with it between those nodes. Such an arrangement is called a ***supernode***. Examples of supernodes are shown in **Fig. 3-4**. Formally:

▶ A ***supernode*** is the combination of two extraordinary nodes (excluding the reference node) between which a voltage source exists. The voltage source may be of the independent or dependent type, and the voltage source may include elements in parallel with it (such as R_6 in parallel with the 16-V source of supernode B in **Fig. 3-4**) but not in series with it. If one of the two nodes of a supernode is a reference (ground) node, it is called a ***quasi-supernode***. ◀



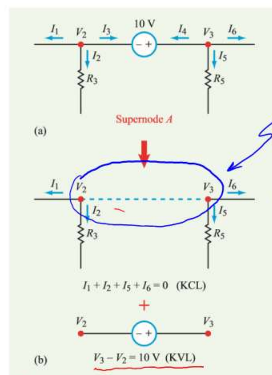


Figure 3-5: A supernode composed of nodes V_2 and V_3 can be represented as a single node, in terms of summing currents flowing out of them, plus an auxiliary equation that defines the voltage difference between V_3 and V_2.

supernode
KCL
$I_1 + I_2 + I_5 + I_6 = 0$

KCL $I_1 + I_2 + I_3 + I_4 = 0$ (1)

$I_1 = \frac{V_1 - 4}{2}$, $I_2 = \frac{V_1}{4}$, $I_3 = \frac{V_2}{8}$, $I_4 = -2$

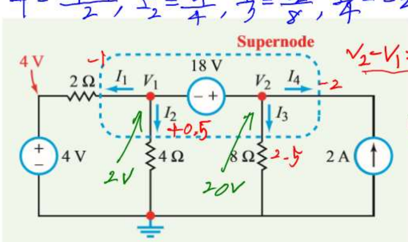


Figure 3-6: Circuit for Example 3-3.

$V_2 - V_1 = 18$ (2)

$V_2 = V_1 + 18$ (2)'

(1) & (2)' →

$\frac{V_1 - 4}{2} + \frac{V_1}{4} + \frac{V_1 + 18}{8} - 2 = 0$

$\Rightarrow 4V_1 - 16 + 2V_1 + V_1 + 18 - 16 = 0$

$7V_1 - 14 = 0$ $\boxed{V_1 = 2}$

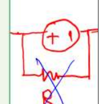
Supernode Attributes

(1) At a supernode, Kirchhoff's current law (KCL) can be applied to the combination of the two nodes as if they are a single node, but the two nodes retain their own identities.

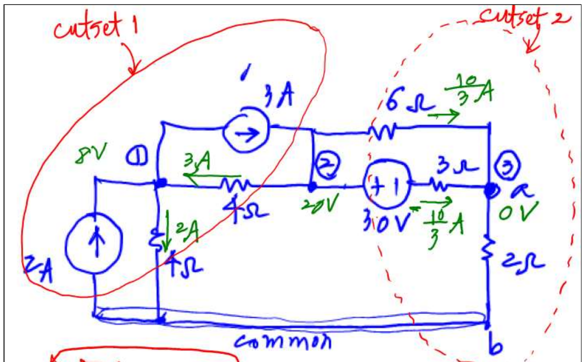
(2) Kirchhoff's voltage law (KVL) is used to express the voltage difference between the two nodes in terms of the voltage of the source between them. This provides the ***supernode auxiliary equation***.

(3) If a supernode contains a resistor in parallel with the voltage source, the resistor exercises no influence on the currents and voltages in the other parts of the circuit, and therefore, it may be ignored altogether.

(4) For a quasi-supernode, the node-voltage of the non-reference node is equal to the voltage magnitude of the source.



Ignore



$$\boxed{\sum_{cutset} I_k = 0}$$ generalized version of supernode.

Mesh Analysis

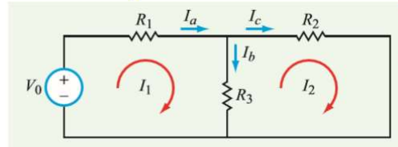


Figure 3-7: Circuit containing two meshes with mesh currents I_1 and I_2.

$I_a = I_1$

$I_c = I_2$

$I_b = I_1 - I_2$

KVL mesh 1 $-V_0 + R_1 I_1 + R_3(I_1 - I_2) = 0$

mesh 2 $R_3(I_2 - I_1) + R_2 I_2 = 0$

$$\begin{bmatrix} R_1+R_3 & -R_3 \\ -R_2 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \end{bmatrix}$$

solve for I_1, I_2

then I_a, I_b, I_c

Mesh Analysis

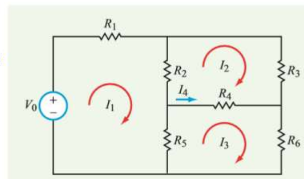


Figure 3-8: Circuit for Example 3-4.

Solution: (a) Applying the symmetry pattern inherent in the structure of the mesh-current equations, we have

$$(R_1 + R_2 + R_5)I_1 - R_2 I_2 - R_5 I_3 = V_0 \qquad \text{(mesh 1)}, \quad (3.15a)$$

$$-R_2 I_1 + (R_2 + R_3 + R_4)I_2 - R_4 I_3 = 0 \qquad \text{(mesh 2)}, \quad (3.15b)$$

and

$$-R_5 I_1 - R_4 I_2 + (R_4 + R_5 + R_6)I_3 = 0 \qquad \text{(mesh 3)}. \quad (3.15c)$$

$$\begin{bmatrix} R_1+R_2+R_5 & -R_2 & -R_5 \\ -R_2 & R_2+R_3+R_4 & -R_4 \\ -R_5 & -R_4 & R_4+R_5+R_6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_0 \\ 0 \\ 0 \end{bmatrix}$$

Exercise 3-5: Determine the current I in the circuit of Fig. E3.5.

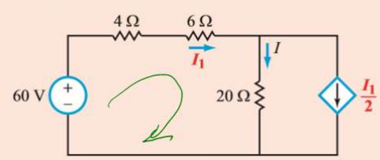


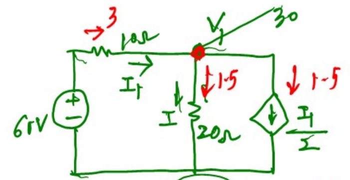
Figure E3.5

Answer: $I = 1.5$ A. (See CAD)

$I_1 = I + \frac{I_1}{2}$

$I = \frac{I_1}{2}$ or $I_1 = 2I$

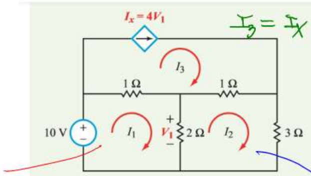
KVL $-60 + (4+6)2I + 20I = 0 \Rightarrow \underline{I = 1.5}$



KCL : $\dfrac{60-V_1}{10} = \dfrac{V_1}{20} + \dfrac{60-V_1}{10 \times 2}$

×20 $\quad 120 - 2V_1 = \cancel{V_1} + 60 - \cancel{V_1}$

$2V_1 = 120 - 60 \Rightarrow \boxed{V_1 = 30}$



$I_3 = I_x = 4V_1 = 8(I_1 - I_2)$

$V_1 = 2(I_1 - I_2)$

Mesh1

$-10 + 1(I_1 - I_3) + 2(I_1 - I_2) = 0$

$\Downarrow$

$\begin{cases} -10 + I_1 \\ -8I_1 + 8I_2 \\ +2I_1 - 2I_2 = 0 \end{cases}$

Mesh2

$2(I_2 - I_1) + 1(I_2 - I_3) + 3I_2 = 0$

$\begin{cases} 2I_2 - 2I_1 \\ +I_2 - 8I_1 + 8I_2 \\ +3I_2 = 0 \end{cases}$

Figure 3-9: Mesh-current solution for a circuit containing a dependent source (Example 3-5).

Hence,

$$I_3 = 4V_1 = 8(I_1 - I_2). \qquad (3.17)$$

After inserting Eq. (3.17) into Eqs. (3.16a and b) and collecting terms in I_1 and I_2, we end up with

$$-5I_1 + 6I_2 = 10,$$
$$-10I_1 + 14I_2 = 0.$$

Solution of this pair of simultaneous equations gives

$$I_1 = -14\text{ A}, \qquad I_2 = -10\text{ A}.$$