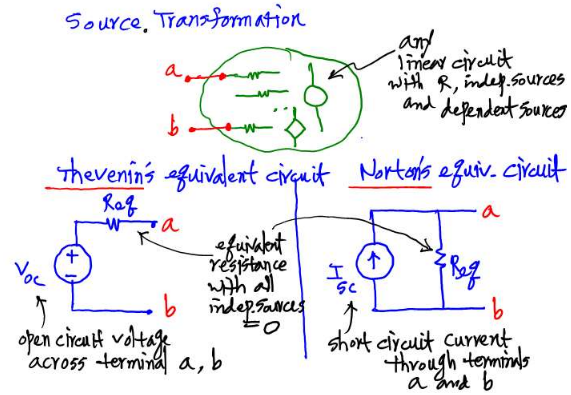
EE101 Lect 5 Jan 22, 2019

Linear vs. nonlinear circuits (eBook pp 86 – 102)

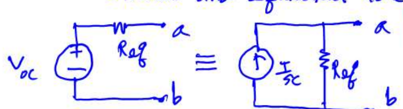
HW #3 (for Quiz 3 on Jan 29)

1. Prob. 3.3
2. Prob. 3.7
3. Prob. 3.11
4. Prob. 3.20
5. Prob. 3.26
6. Prob. 3.32
7. Prob. 3.48
8. Prob. 3.58

Source Transformation



Fact 1. Thevenin's and Norton's equivalent circuits are equivalent to each other.



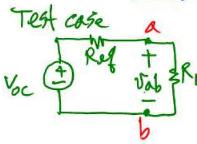
$I_{sc} = \frac{V_{oc}}{R_{eq}}$

$V_{oc} = R_{eq} I_{sc}$

$R_{eq} = \frac{V_{oc}}{I_{sc}}$

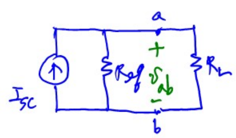
Example, For any (load) resistor connected between a and b will draw same current through it and also voltage across it.

Test case



$I_{ab} = \frac{V_{oc}}{R_{eq} + R_L}$

$V_{ab} = V_{oc} \frac{R_L}{R_{eq} + R_L}$



$V_{ab} = I_{sc} (R_{eq} \| R_L)$

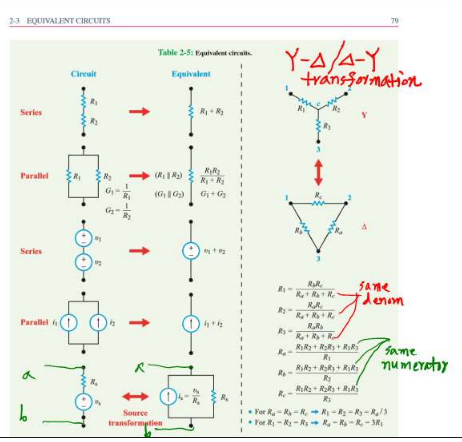
$= I_{sc} \frac{R_L R_{eq}}{R_{eq} + R_L}$

$I_{sc} = \frac{V_{oc}}{R_{eq}}$, thus

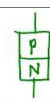
$V_{ab} = \frac{V_{oc}}{R_{eq}} \frac{R_L R_{eq}}{R_{eq} + R_L}$

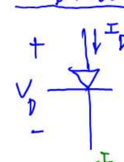
$= V_{oc} \frac{R_L}{R_{eq} + R_L}$ (same) ✓

$I_{ab} = \frac{v_{ab}}{R_L} = \frac{V_{oc}}{R_{eq} + R_n}$ (same) ✓



Diode





$I_D = I_0 \left(e^{\frac{qV_D}{kT}} - 1\right) = I_0 \left(e^{\frac{V_D}{V_T}} - 1\right)$

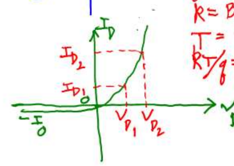
$= I_0 \left(e^{kV_D} - 1\right) = f(V_D)$

$q = 1.6 \times 10^{-19}$ coulomb, $I_0 = 1 \sim 5 \times 10^{-12} A/(\mu m)^2$

k = Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

T = Temperature (°F) = 300°K at room temp

$kT/q = 1.38 \times 10^{-23} \times 300 / 1.6 \times 10^{-19} = 25.88$ [mV]



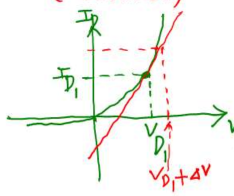
Linearity check for $f(\cdot)$

$y_1 = f(x_1)$, $y_2 = f(x_2)$

$f(\alpha x_1 + \beta x_2) \stackrel{?}{=} \alpha y_1 + \beta y_2$

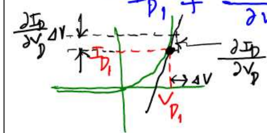
$$f(\alpha V_{D_1} + \beta V_{D_2}) = I_0\left(e^{\alpha V_{D_1} + \beta V_{D_2}} - 1\right)$$
$$= I_0\left(e^{\alpha K V_{D_1}} \cdot e^{\beta K V_{D_2}} - 1\right)$$
$$\neq \alpha \underbrace{I_0\left(e^{K V_{D_1}} - 1\right)}_{I_{D_1}} + \beta \underbrace{I_0\left(e^{K V_{D_2}} - 1\right)}_{I_{D_2}}$$

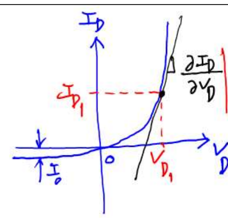
(nonlinear)



$$I_{D_1} = I_0\left(e^{K V_{D_1}} - 1\right)$$
For $V_D = V_{D_1} + \Delta V$
$$I_D = I_0\left(e^{K(V_{D_1} + \Delta V)} - 1\right)$$
$$= I_0\left(e^{K V_{D_1}} e^{K \Delta V} - 1\right)$$

$$= I_0\left(e^{K V_{D_1}}\left(1 + K\Delta V + \frac{(K\Delta V)^2}{2!} + \cdots\right) - 1\right)$$
$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$
$$= I_0\left(e^{V_{D_1}} - 1 + e^{K V_{D_1}} K \Delta V + \cdots\right)$$
≈ 0 for $\Delta V \ll 1$
$$\simeq I_{D_1} + \underbrace{K I_0 e^{K V_{D_1}}}\,\Delta V$$
$$= I_{D_1} + \frac{\partial I_D}{\partial V_D}\Delta V$$



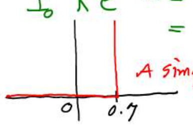


$$\left.\frac{\partial I_D}{\partial V_D}\right|_{V_{D_1}} = \frac{\partial}{\partial V_D}\left[I_0\left(e^{K V_D} - 1\right)\right]\Bigg|_{V_{D_1}}$$
$$= K I_0 e^{K V_{D_1}}$$
where $K = \frac{1}{kT/q} = \frac{1}{26\,mV}$ (T = room temp.)

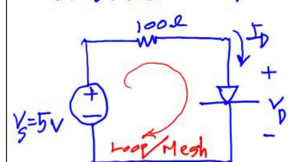
For $V_{D_1} = 0.7V$, $K V_{D_1} = \frac{0.7}{0.026} = 26.9$

$$I_0 K e^{K V_{D_1}} = 26.9 \times 4.927 \times 10^{11}$$
$$= 1.326 \times 10^{13}\, I_0 \approx 13\,[A] \text{ huge!}$$

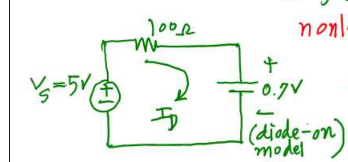
A simple diode model ($I_0 = 1$ PA case)



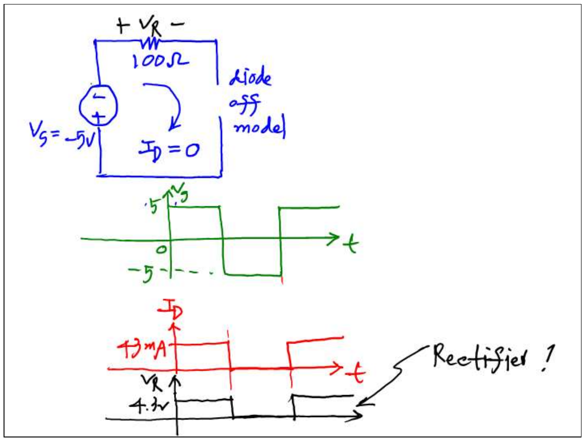
A Diode Circuit

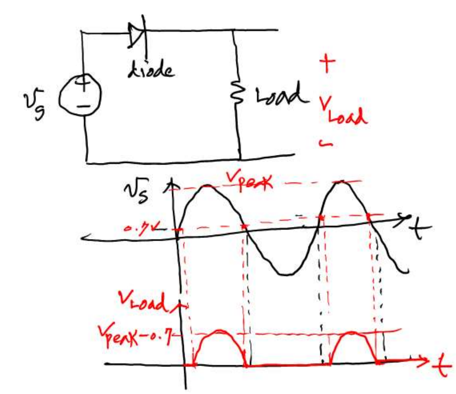


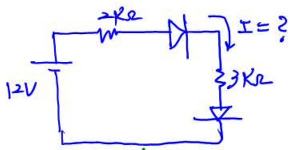
KVL
$$-5 + 100 I_D + V_D = 0$$
$$I_D = I_0\left(e^{K V_D} - 1\right)$$
$$\Rightarrow 5 = 100 I_0\left(e^{K V_D} - 1\right) + V_D = 0$$
nonlinear eq !



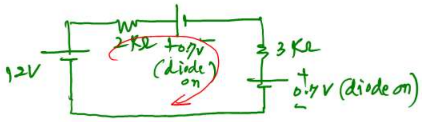
$$I_D = \frac{5 - 0.7}{100} = \frac{4.3}{100} A$$
$$= 43\ mA$$







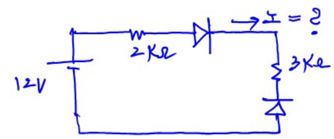
solution



KVL : $-12 + 2K\,I + 0.7 + 3K\,I + 0.7 = 0$

$5K\,I = 12 - 0.7 - 0.7 = 10.6$

$I = \frac{10.6}{5K} = 2.12$ [mA] (ans)



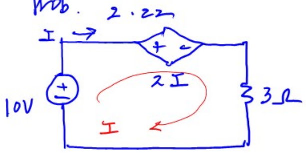
If $I > 0$, diode 1 is on, but diode 2 off

If $I < 0$, diode 1 off, diode 1 on

If $I = 0$ both diodes off

⇒ In all cases, the circuit flows no current $I = 0$ (ans)

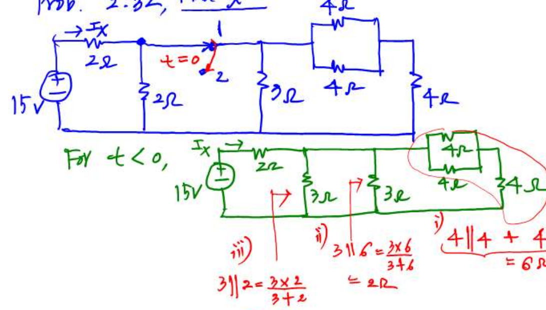
Prob. 2.22



KVL $-10 + 2I + 3 \times I = 0$

$10 = 5I$, $I = \frac{10}{5} = 2A$

Prob. 2.32, Find I_x



i) $4 \| 4 + 4 = 6\Omega$

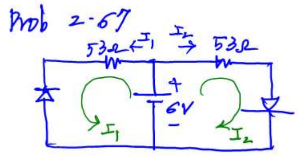
ii) $3 \| 6 = \frac{3 \times 6}{3+6} = 2\Omega$

iii) $3 \| 2 = \frac{3 \times 2}{3+2} = 1.2\Omega$

$I_x = \frac{15}{2+1.2} = \frac{15}{3.2} = 0.469A$

For $t > 0$, $I_x = \frac{15}{2+2} = \frac{15}{4} = 3.75A$

Prob 2-67



Right side mesh : KVL $-6 + 53I + 0.7 = 0$

$I = \frac{6-0.7}{53} = 0.1A$

Left side mesh : KVL ? But diode is off !

$I_1 = 0$

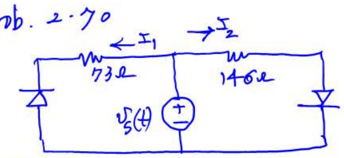
(If $I_1 < 0$, then KVL counterclockwise,

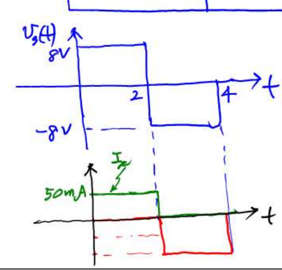
⇒ $+0.7 + 53I + 6 = 0$

$I = -6.7/53 = -0.126A$?? X

this will violate KVL in the outer loop (not right)

Prob. 2.70





• For $0 < t < 2$

$I_2 = \frac{8-0.7}{146} = \frac{7.3}{146} = 50$ [mA]

$I_1 = 0$

• For $2 < t < 4$

$I_1 = -(8-0.7)/73 = -100$ [mA]

$I_2 = 0$