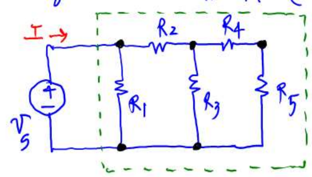
EE101 Lect 4 Jan 17, 2019

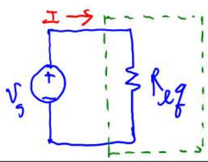
Equivalent Circuits (textbook pp 70-86)



we denote serial connection by + and parallel connection by ||, then

$R_{eq} = \{ [(R_4 + R_5) \,||\, R_3] + R_2 \} \,||\, R_1$

should start from the far end



Again, I [A] Ampere

V [V] Volt

R [Ω] ohm

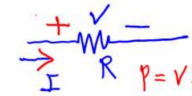
Ohm's Law

$V = RI$

[V] [Ω][A]

$I\,[A] = \dfrac{V\,[V]}{R\,[\Omega]}$

$R\,[\Omega] = \dfrac{V\,[V]}{I\,[A]}$



$P = VI$

$P = -V_s I$

V_s (voltage) Source

Conductance Unit [S] siemens

For a conducting element, electrical resistance *R* and electrical conductance *G* are defined as

G = \frac{1}R = \frac{I}V

$G = \dfrac{1}{R} = \dfrac{I\,[A]}{V\,[V]}$ [S]

where *I* is the electric current through the object and *V* is the voltage (electrical potential difference) across the object.

The unit **siemens** for the conductance *G* is defined by

\mathrm{S} = \Omega^{-1} = \dfrac{\mathrm{A}}{\mathrm{V}}

$[S] = [\Omega]^{-1}$

where Ω is the ohm, A is the ampere, and V is the volt.

For a device with a conductance of one siemens, the electric current through the device will increase by one ampere for every increase of one volt of electric potential difference across the device.

The conductance of a resistor with a resistance of five ohms, for example, is $(5\ \Omega)^{-1}$, which is equal to 200 mS.

5Ω Resistance

conductance $G = \dfrac{1}{R} = \dfrac{1}{5\,[\Omega]} = 0.2\,S = 200\,mS$

Ohm mho

Mho[edit] [℧] as $\left[\dfrac{1}{\Omega}\right]$

"Mho" redirects here. It is not to be confused with Mohs (disambiguation).

A name that is used as an alternative to the *siemens* is the ***mho*** /moʊ/, the reciprocal of one ohm. It is derived from spelling *ohm* backwards and is written as an upside-down capital Greek letter omega: mho, Unicode symbol U+2127 (℧). According to Maver[2] the term *mho* was suggested by Sir William Thomson (Lord Kelvin). The *mho* was officially renamed to the *siemens*, replacing the old meaning of the "*siemens* unit", at a conference in 1881.[3]

NIST's *Guide for the Use of the International System of Units (SI)* refers to the mho as an "unaccepted special name for an SI unit", and indicates that it should be strictly avoided.[4]

Avg = 5.73

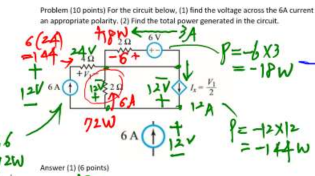
σ = 1.93

EE101 Winter 2019 Quiz #1

January 15, 2019

Name________________ Student ID________________

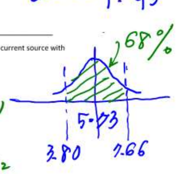
Problem (10 points) For the circuit below, (1) find the voltage across the 6A current source with an appropriate polarity. (2) Find the total power generated in the circuit.



$P = -6 \times 3 = -18W$

$-12 \times 6 = -72W$

$P = -12 \times 12 = -144W$



Answer (1) (6 points)

Voltage = 12 [V]

Mark the Polarity to the source figure with + and − signs.

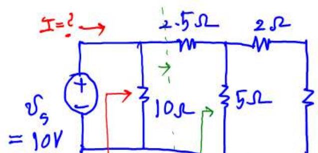
144 + 18 + 72 = 234W

Σ = 0

Answer (2) (4 points)

Total power generated in the circuit = 234 [W].

72 + 18 + 144 = 234 W (−)



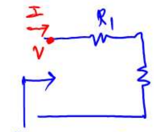
$R_{eq} = \{ [(2+3) \,||\, 5] + 2.5 \} \,||\, 10$

$\dfrac{5 \times 5}{5+5} = 2.5$

5

$5 \,||\, 10 = \dfrac{5 \times 10}{5 + 10} = \dfrac{50}{15} = \dfrac{10}{3}\ \Omega$

$I = \dfrac{10}{(10/3)} = 3\,A$

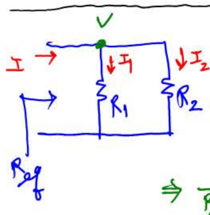


(serial connection)

$V = R_{eq} I = R_1 I + R_2 I$
$= (R_1 + R_2) I$

$R_{eq} = R_1 + R_2$

- $R_{eq} = R_1 + R_2$



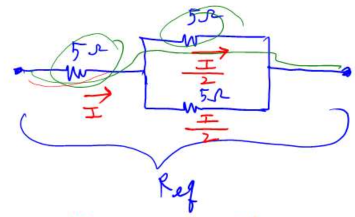
(parallel connection)

$I = I_1 + I_2$

$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$

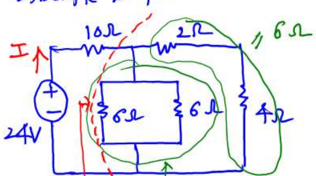
- $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2$



$R_{eq} = 5 + \underbrace{5 \| 5}_{\frac{5 \times 5}{5+5} = 2.5} = 7.5\,\Omega$

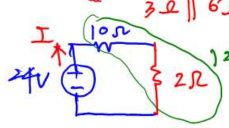
not 10Ω

Example 2-12



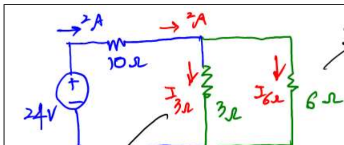
$6 \| 6 = \frac{6 \times 6}{6+6} = 3\,\Omega$

$3\,\Omega \| 6\,\Omega = \frac{3 \times 6}{3+6} = 2\,\Omega$

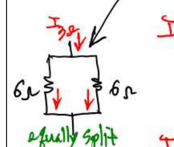


$I = \frac{24V}{12\,\Omega} = 2A$

$P_{24V} = -24V \times 2A = -48W$ (supplied)



$I_{2\Omega} = I_{4\Omega} = \frac{2}{3} A$

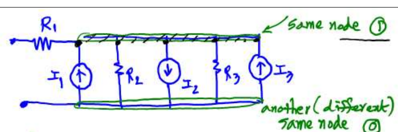


current division based on conductances $\left(G_K = \frac{1}{R_K}\right)$

$I_{3\Omega} = 2A\left(\frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}}\right) = 2A\left(\frac{2}{2+1}\right) = \frac{4}{3} A$

$I_{6\Omega} = 2A\left(\frac{\frac{1}{6}}{\frac{1}{3} + \frac{1}{6}}\right) = 2A\left(\frac{1}{2+1}\right) = \frac{2}{3} A$

$I_{3\Omega} + I_{6\Omega} = \frac{4}{3} + \frac{2}{3} = 2A$ ✓

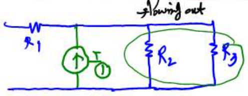


$V_{①}$

Total current flowing into node ① :

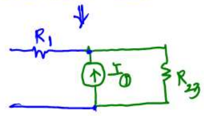
$I_{①} = I_1 + (-I_2) + I_3$

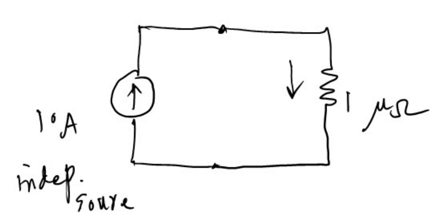
flowing out

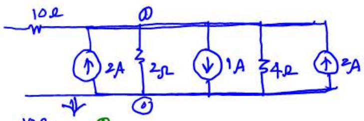


$= R_2 \| R_3 = \frac{R_2 \cdot R_3}{R_2 + R_3} = R_{23}$

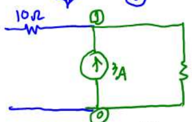
⇓





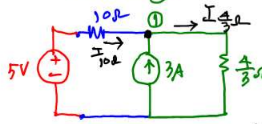


10Ω ① 2A 2Ω 1A 4Ω 2A ⓪



10Ω ① 3A ⓪

$2\Omega \| 4\Omega = \frac{2\times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\Omega$



5V 10Ω ① $I_{10\Omega}$ $I_{\frac{4}{3}\Omega}$ 3A $\frac{4}{3}\Omega$

KCL at ①

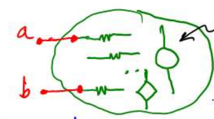
$$I_{10\Omega} + 3A = I_{\frac{4}{3}\Omega}$$

$$\frac{5-V_1}{10} + 3 = \frac{V_1}{\left(\frac{4}{3}\right)} = \frac{3V_1}{4}$$

$\times 20$ on both sides $10 - 2V_1 + 3\times 20 = 15V_1$

$\Rightarrow 70 = 17V_1 \Rightarrow V_1 = 70/17\,[V]$

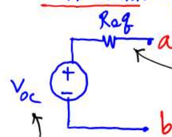
Source Transformation



a b

any linear circuit with R, indep. sources and dependent sources

Thevenin's equivalent circuit

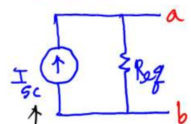


R_{eq} a V_{oc} b

equivalent resistance with all indep. sources $= 0$

open circuit voltage across terminal a, b

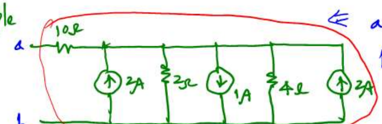
Norton's equiv. circuit



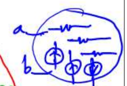
a I_{sc} R_{eq} b

short circuit current through terminals a and b

Example

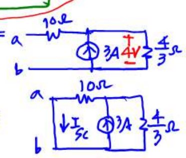


a 10Ω 2A 2Ω 1A 4Ω 2A b



a b

Thevenin's equivalent circuit



$= $ a 10Ω 3A 4V $\frac{4}{3}\Omega$ b

a 10Ω I_{sc} 3A $\frac{4}{3}\Omega$ b

$V_{oc} = ?$ $\quad 3A \times \frac{4}{3}\Omega = 4V$

$R_{eq} = ? = \frac{V_{oc} = 4V}{I_{sc} = \frac{6}{17}}$ $\quad I_{sc} = ?$

$= \frac{4\times 17}{6}$

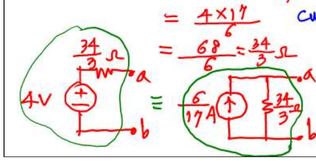
$= \frac{68}{6} = \frac{34}{3}\Omega$

current division

$$I_{sc} = 3A\,\frac{\frac{1}{10}}{\frac{1}{10} + 1/\left(\frac{4}{3}\right)}$$

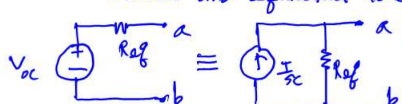
$$= 3A\,\frac{2}{2+15} = \frac{6}{17}A$$

$\times 20$ both sides



$\frac{34}{3}\Omega$ a 4V b $\equiv$ $\frac{6}{17}A$ $\frac{34}{3}\Omega$ a b

Fact 1. Thevenin's and Norton's equivalent circuits and equivalent to each other.



V_{oc} R_{eq} a b $\equiv$ I_{sc} R_{eq} a b

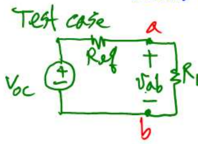
$I_{sc} = \frac{V_{oc}}{R_{eq}}$

$V_{oc} = R_{eq} I_{sc}$

$R_{eq} = \frac{V_{oc}}{I_{sc}}$

Example, For any (load) resistor connected between a and b will draw same current through it and also voltage across it.

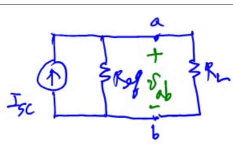
Test case



V_{oc} R_{eq} a V_{ab} R_L b

$$I_{ab} = \frac{V_{oc}}{R_{eq} + R_L}$$

$$V_{ab} = V_{oc}\,\frac{R_L}{R_{eq} + R_L}$$



I_{sc} R_{eq} a V_{ab} b R_L

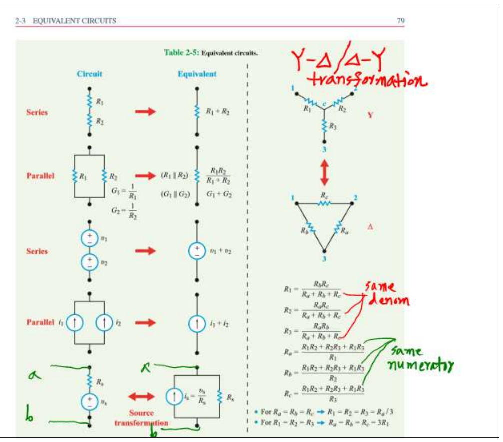
$$V_{ab} = I_{sc}\left(R_{eq} \| R_L\right) = I_{sc}\,\frac{R_L R_{eq}}{R_{eq} + R_L}$$

$I_{sc} = \frac{V_{oc}}{R_{eq}}$, thus

$$V_{ab} = \frac{V_{oc}}{R_{eq}}\,\frac{R_L R_{eq}}{R_{eq} + R_L}$$

$$= V_{oc}\,\frac{R_L}{R_{eq} + R_L} \text{ (same)}$$

$$I_{ab} = \frac{V_{ab}}{R_L} = \frac{V_{oc}}{R_{eq} + R_L} \text{ (same)}$$



$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$

$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$

$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$

$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$

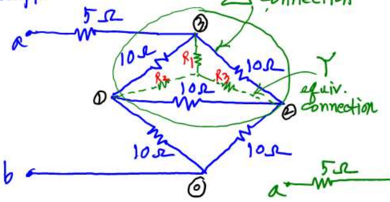
$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$

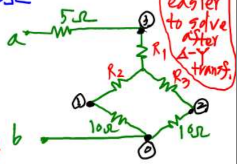
$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$

- For $R_a = R_b = R_c \rightarrow R_1 = R_2 = R_3 = R_a/3$
- For $R_1 = R_2 = R_3 \rightarrow R_a = R_b = R_c = 3R_1$

Y-Δ / Δ-Y transformation
(Wye-Delta)/(Delta-Wye)

Example

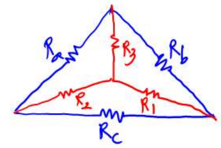




Much easier to solve after Δ-Y transf.

Then how do we find R_1, R_2, R_3?

Δ-Y transformation formula



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

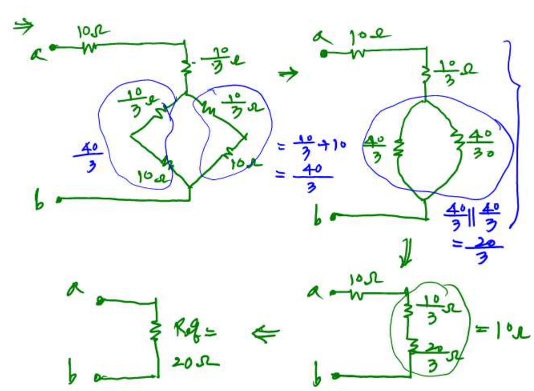
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

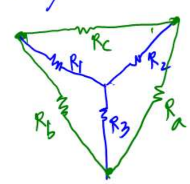
same denom

In the previous example, $R_a = R_b = R_c = 10\Omega$

thus $R_1 = \frac{10\times 10}{10+10+10} = \frac{100}{30} = \frac{10}{3} = R_2 = R_3$



Conversely Y-Δ transformation

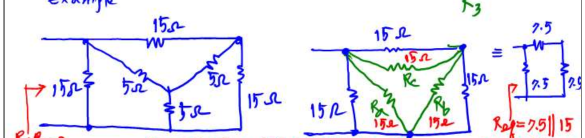


$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad \text{(same)}$$

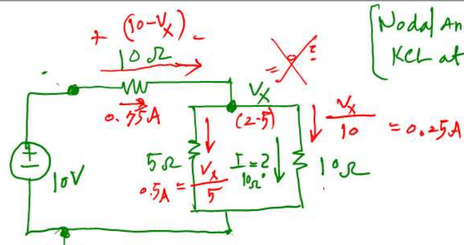
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad \text{(same)}$$

Example



$R_a = R_b = R_c = \frac{5\times5 + 5\times5 + 5\times5}{5} = \frac{75}{5} = 15$

$R_{eq} = 7.5 \| 15 = \frac{7.5\times15}{7.5+15} = 5\Omega$

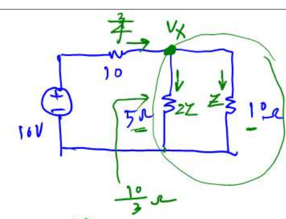


Nodal Analysis
KCL at (x)

$$\frac{10-V_x}{10} = \frac{V_x}{5} + \frac{V_x}{10}$$

$\times 10 \qquad 10 - V_x = 2V_x + V_x$

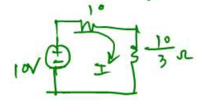
$10 = 4V_x \Rightarrow V_x = 2.5\,V$ ✓



$\frac{3}{4} = 2Z + Z = 3Z$

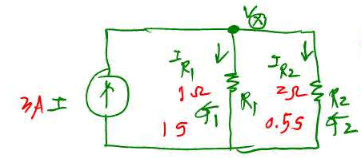
$Z = \frac{1}{4}A = 0.25$

$5 \| 10 = \frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}$



$$I = \frac{10}{10 + \frac{10}{3}} = \frac{10}{\frac{30+10}{3}} = \frac{30}{40} = \frac{3}{4}\,A$$

$V_x = \frac{3}{4}A \times \frac{10}{3}\Omega = 2.5\,[V]$ ✓



$$I_1 = \frac{G_1}{G_1 + G_2} I \qquad \frac{1}{1+0.5} \times 3 = 2$$

$$I_2 = \frac{G_2}{G_1 + G_2} I \qquad \frac{0.5}{1+0.5} \times 3 = 1$$

$$I = I_{R_1} + I_{R_2}$$

$$= \frac{\cancel{V_x}}{R_1} + \frac{\cancel{V_x}}{R_2} \qquad = \cancel{V_x}\,\frac{R_1 + R_2}{R_1 \cdot R_2}$$

$$V_x = I (R_1 \parallel R_2) = I \frac{R_1 R_2}{R_1 + R_2}$$

$$I = V_x \frac{R_1 + R_2}{R_1 R_2}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 \cdot R_2}$$