EE101 Lecture #3 Jan 15, 2019

So far we have discussed

- Elements of electronic circuits
  R, L, C, independent sources,
  dependent sources
- Electrical variables
  voltage (V), current (I), power ($p = VI$)
  Energy ($E(t) = \int_0^t p(\tau)\,d\tau$), Average power
  $P_{avg} = \frac{1}{T}\int^T p(\tau)\,d\tau$
- Loop/Mesh currents, Node voltages
- KCL, KVL, Ohm's Law

HW #2 for Quiz on Jan 22

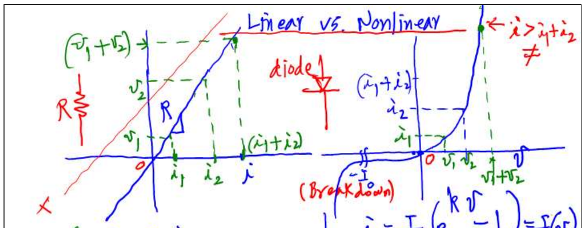
[1] Prob. 2.11
[2] Prob. 2.19
[3] Prob. 2.25
[4] Prob. 2.28
[5] Prob. 2.33
[6] Prob. 2.40
[7] Prob. 2.43
[8] Prob. 2.47

DRC Quiz is in E2-215, 234 at 2:40-3:10

Group Tutor Hours in E2-516

Tu, Th 3:30 – 6 pm
W 3 – 4 pm

Linear vs. Nonlinear



$v = R i$

$v_1 = R i_1$ $v_1 = f(i_1)$
$v_2 = R i_2$ $v_2 = f(i_2)$

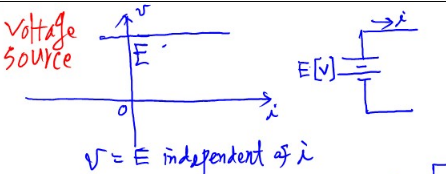
$f(\alpha i_1 + \beta i_2) = \alpha R i_1 + \beta R i_2$
$= \alpha f(i_1) + \beta f(i_2)$

$f(\cdot)$ is Linear

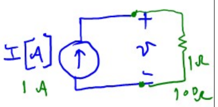
$i = I_0(e^{kv} - 1) = f(v)$
$f(\alpha v_1 + \beta v_2)$
$= I_0(e^{k(\alpha v_1 + \beta v_2)} - 1)$
$> \alpha f(v_1) + \beta f(v_2)$

$f(\cdot)$ is Non-Linear

Voltage Source

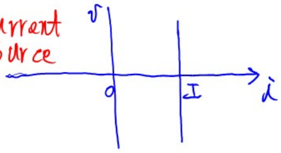


$v = E$ independent of i



$v \neq R i$ (nonlinear)

Current Source

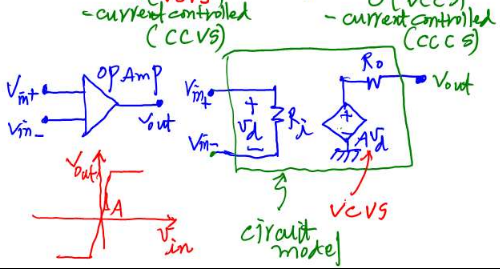


$i = I$ independent of v

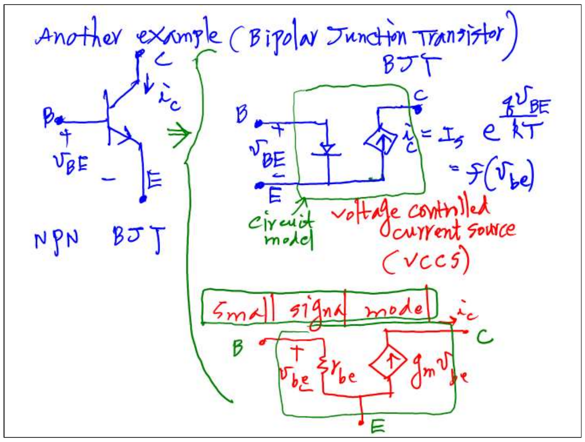
Dependent Sources (4 different types)

dependent voltage source
- voltage controlled (VCVS)
- current controlled (CCVS)

dependent current source
- voltage controlled (VCCS)
- current controlled (CCCS)

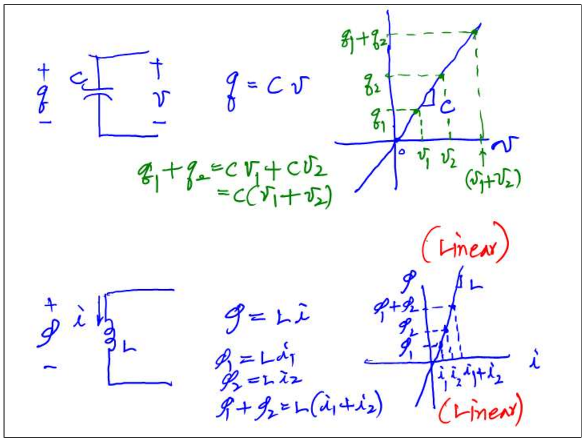


Another example (Bipolar Junction Transistor)



$i_C = I_s e^{\frac{qv_{BE}}{kT}}$

$= f(v_{be})$



$q = Cv$

$q_1 + q_2 = Cv_1 + Cv_2$
$= C(v_1 + v_2)$

$\varphi = Li$

$\varphi_1 = Li_1$
$\varphi_2 = Li_2$
$\varphi_1 + \varphi_2 = L(i_1 + i_2)$

In general

$y = f(x)$ f linear function

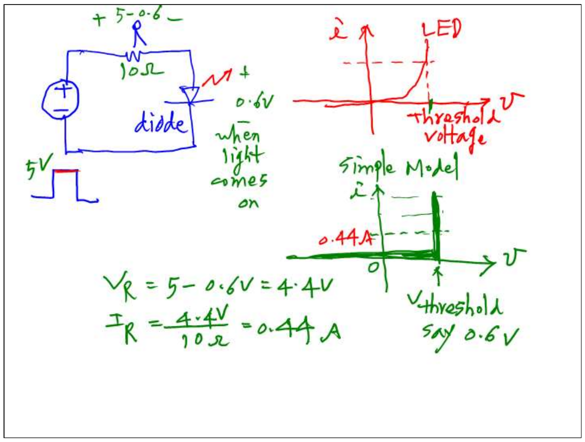
$y_1 = f(x_1)$, $y_2 = f(x_2)$

For $x = ax_1 + bx_2$

$y = f(ax_1 + bx_2)$

$\overset{\uparrow}{=} af(x_1) + bf(x_2) = ay_1 + by_2$

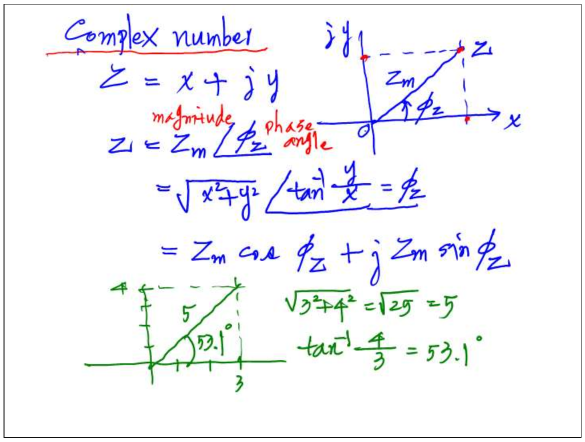
(f linear)



$V_R = 5 - 0.6V = 4.4V$

$I_R = \frac{4.4V}{10\Omega} = 0.44A$

Complex number



$Z = x + jy$

$Z = Z_m \angle \phi_Z$ magnitude phase angle

$= \sqrt{x^2+y^2} \angle \tan^{-1}\frac{y}{x} = \phi_Z$

$= Z_m \cos\phi_Z + jZ_m \sin\phi_Z$

$\sqrt{3^2+4^2} = \sqrt{25} = 5$

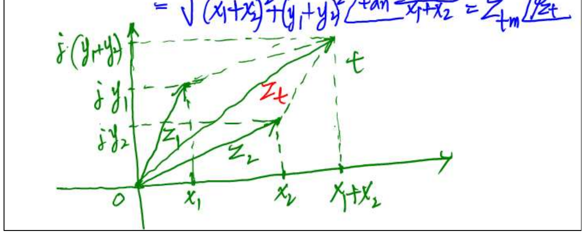
$\tan^{-1}\frac{4}{3} = 53.1°$

$Z_1 = x_1 + jy_1 = Z_{1m} \angle \phi_{Z_1}$

$Z_2 = x_2 + jy_2 = Z_{2m} \angle \phi_{Z_2}$

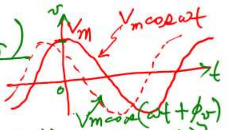
$Z_t = Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$

$= \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2} \angle \tan^{-1}\frac{y_1+y_2}{x_1+x_2} = Z_{tm} \angle \phi_{Z_t}$



$$\underline{v = V_m \cos(\omega t + \phi_v)}$$

$$= \mathrm{Re}\left[V_m e^{j(\omega t + \phi_v)}\right]$$



$$\underset{\uparrow}{\overset{}{=}} \mathrm{Re}\left[V_m\left(\cos(\omega t + \phi_v) + j\sin(\omega t + \phi_v)\right)\right]$$

$$\left(e^{j\theta} = \cos\theta + j\sin\theta\right)$$

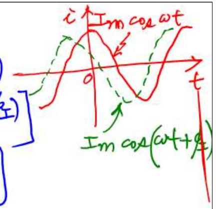
$$= \mathrm{Re}\left[V_m e^{j\phi_v} e^{j\omega t}\right] = \mathrm{Re}\left[\mathbb{V} e^{j\omega t}\right]$$

$$\boxed{\mathbb{V} = V_m e^{j\phi_v} = V_m \angle\phi_v}$$

phasor

Likewise

$$I = I_m \cos(\omega t + \phi_I)$$



$$= \mathrm{Re}\left[I_m e^{j(\omega t + \phi_I)}\right]$$

$$= \mathrm{Re}\left[I_m e^{j\phi_I} e^{j\omega t}\right]$$

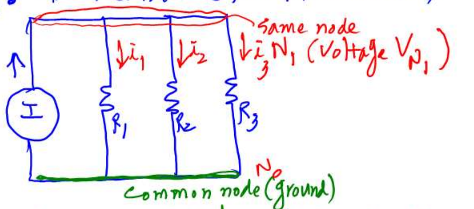
$$= \mathrm{Re}\left[\mathbb{I} e^{j\omega t}\right]$$

$$\boxed{\mathbb{I} = I_m e^{j\phi_I} = I_m \angle\phi_I}$$

phasor

$$\mathbb{V}\cdot\mathbb{I} = V_m e^{j\phi_V}\cdot I_m e^{j\phi_I} = V_m I_m e^{j(\phi_V + \phi_I)}$$

[Ex]. Linear Resistors (Parallel Connection)

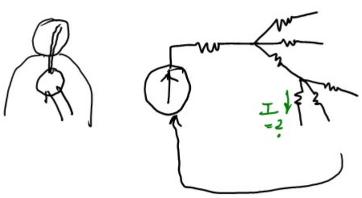


In this case Nodal Analysis is better than Loop Analysis. Ohm's Law

$$I = i_1 + i_2 + i_3 \underset{\downarrow}{=} \frac{V_{N_1}}{R_1} + \frac{V_{N_1}}{R_2} + \frac{V_{N_1}}{R_3}$$

$$= V_{N_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\underline{V_{N_1} = I \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$



In Ohm's Law $v = R\,i$, R = resistance [Ω] Ohm

or $i = \frac{v}{R} = \frac{1}{R} v$

$= G v$

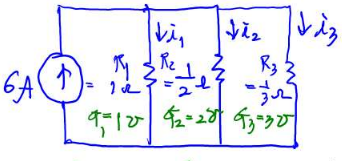
G is conductance [℧] mho

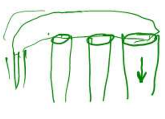
$$\text{So } V_{N_1} = I \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= I \big/ (G_1 + G_2 + G_3)$$

$$i_1 = \frac{V_{N_1}}{R_1} = G_1 V_{N_1} = I\left[\frac{G_1}{G_1 + G_2 + G_3}\right]$$

Revisit of the Example



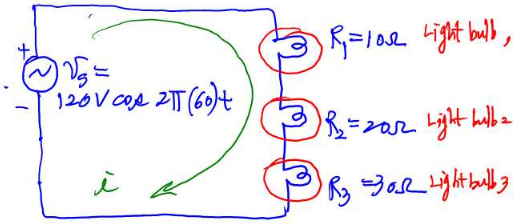


$$i_1 = I\frac{G_1}{G_1 + G_2 + G_3} = 6\frac{1}{1+2+3} = 1\,[A]$$

$$i_2 = I\frac{G_2}{G_1 + G_2 + G_3} = 6\frac{2}{1+2+3} = 2\,[A]$$

$$i_3 = I\frac{G_3}{G_1 + G_2 + G_3} = 6\frac{3}{1+2+3} = 3\,[A]$$

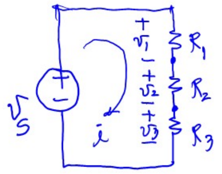
$I = i_1 + i_2 + i_3$ (✓) (KCL)



Loop (Mesh) current $i = ?$

By Ohm's Law $i = \frac{v_s}{R_1+R_2+R_3} = \frac{120\cos 120\pi t}{60}$

$= 2\cos 120\pi t$ [A]



KVL

$-v_s + v_1 + v_2 + v_3 = 0$

$\Rightarrow v_s = v_1 + v_2 + v_3$

$= iR_1 + iR_2 + iR_3$

$= i(R_1 + R_2 + R_3)$

$$i = \frac{v_s}{R_1+R_2+R_3}$$

$$\left.\begin{aligned} v_1 &= iR_1 = v_s\frac{R1}{R_1+R_2+R_3} \\ &\vdots \\ v_3 &= iR_3 = v_s\frac{R_3}{R_1+R_2+R_3} \end{aligned}\right\} v_K = v_s\frac{R_K}{\Sigma R_i}$$

voltage division

Average power in Light Bulb 1

$$P_1 = \frac{1}{T}\int_0^T v_1(t)\,i(t)\,dt$$

$$= \frac{1}{T}\int_0^T R_1\, i^2(t)\,dt$$

$v_1 = R_1 i$

$$= \frac{10}{T}\int_0^T \left[2\cos 120\pi t\right]^2 dt$$

$R_1 = 10\Omega$

$$= \frac{10\times 4}{T}\int_0^T \cos^2 120\pi t\,dt$$

$$= \frac{40}{T}\int_0^T \frac{1}{2}\left(1+\cos 2(120\pi t)\right)dt$$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$$= \frac{40}{T}\left[\frac{1}{2}T + \underbrace{\frac{1}{2}\int_0^T \cos 2(120\pi t)\,dt}_{=0}\right]$$

$= 20$ W

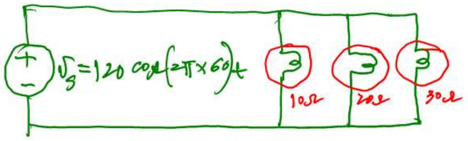
For Ligh Bulb 2 ($R_2 = 20\Omega$)

$P_2 = \frac{20}{10}[20W] = 40W$

$P_3 = \frac{30}{10}[20W] = 60W$

Total wattage $P_1 + P_2 + P_3 = \boxed{120W}$

If bulbs are connected in Parallel



Avg Power in bulb 1

$$P_1 = \frac{1}{T}\int_0^T v_s(t)\,i_1(t)\,dt = \frac{1}{T}\int_0^T \frac{v_s^2(t)}{R_1}dt$$

$$= \frac{1}{10 \ T}\int_0^T \left[120\cos 120\pi t\right]^2 dt$$

$$= \frac{14400}{10T}\left[\frac{1}{2}\int_0^T (1+\cos 240\pi t)\,dt\right]$$

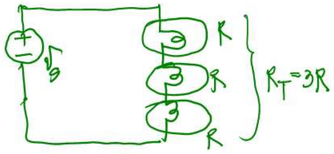
$$= \frac{144}{T}\left[\frac{1}{2}T + \underbrace{\frac{1}{2}\int_0^T \cos 240\pi t\,dt}_{=0}\right]$$

$= 72W$

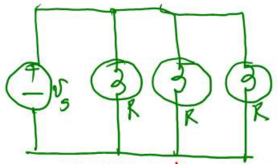
$P_2 = 72W\left(\frac{10}{20}\right) = 36W$ $\left(P(t) = \frac{v_s^2(t)}{R}\right)$

$P_3 = 72W\left(\frac{10}{30}\right) = 24W$

$P_1 + P_2 + P_3 = 72 + 36 + 24 = \boxed{132W}$



$$P(t) = \frac{v_s^2(t)}{3R}$$



$$P(t) = \frac{v_s^2(t)}{R}\times 3$$

9 time more wattage!

(Light Bulb at home are connected this way)

But if too many in ~~Parallel~~ way, then overload (shut off)