ECE 101 Lecture #2 Jan 10, 2019

In Lect #1,

we covered

- circuit elements R, L, C
  sources, dependent sources
- Kirchhoff Current Law (KCL)
  Kirchhoff Voltage Law (KVL)
- Power generation $V \cdot I < 0$
  consumption $V \cdot I > 0$

Conservation of power $\sum P_k = 0$

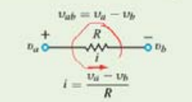
2-1.2 i–υ Characteristics of Ideal Resistor

Based on the results of his experiments on the nature of conduction in circuits, German physicist Georg Simon Ohm (1787–1854) formulated in 1826 the i–υ relationship for a resistor, which has become known as ***Ohm's law***. He discovered that the voltage υ across a resistor is directly proportional to the current i flowing through it, namely

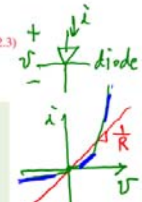
$$\upsilon = iR, \qquad (2.3)$$

with the resistance R being the proportionality factor.

► In compliance with the passive sign convention, current enters a resistor at the "+" side of the voltage across it.



An ideal ***linear resistor*** is one whose resistance R is constant and independent of the magnitude of the current flowing through it, in which case its ***i–υ response*** is a straight line (Fig. 2-4(a)).



Prof Ohm's Stature at Tech U Munchen (2018)

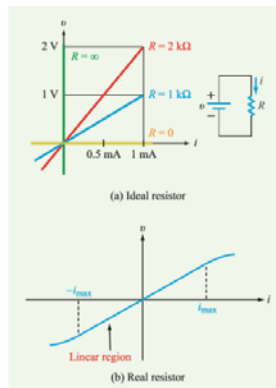


Figure 2-4: i–υ responses of ideal and real resistors.

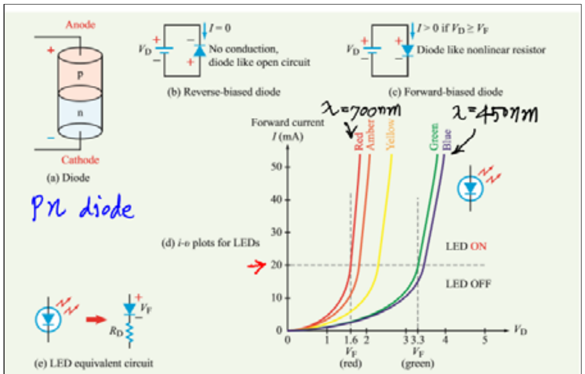


Figure 2-8: p-n junction diode (a) configuration, (b) reverse biased, (c) forward biased, (d) typical i–υ plots for LEDs, and (e) LED equivalent circuit.

KCL

2-2 KIRCHHOFF'S LAWS

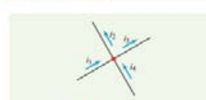


Figure 2-9: Currents at a node.

$i_1 + i_4 = i_2 + i_3$

$i_1 + i_4 - i_2 - i_3 = 0$

$-i_1 - i_4 + i_2 + i_3 = 0$

Mathematically, KCL can be expressed by the compact form:

$$\sum_{n=1}^{N} i_n = 0 \quad \text{(KCL)}, \qquad (2.8)$$

where N is the total number of branches connected to the node, and i_n is the nth current.

► A common convention is to assign a positive "+" sign to a current if it is entering the node and a negative "−" sign if it is leaving it. ◄

$\sum_j i_j = \sum_k i_k$

entering leaving

For the node in Fig. 2-9, the sum of currents entering the node is

$$i_1 - i_2 - i_3 + i_4 = 0, \qquad (2.9)$$

where currents i_1 and i_4 were assigned positive signs because they are labeled in the figure as entering the node, and i_2 and i_3 were assigned negative signs because they are leaving the node.

► A common convention is to assign a positive "+" sign to a current if it is entering the node and a negative "−" sign if it is leaving it. ◄

For the node in Fig. 2-9, the sum of currents entering the node is

$$i_1 - i_2 - i_3 + i_4 = 0, \qquad (2.9)$$

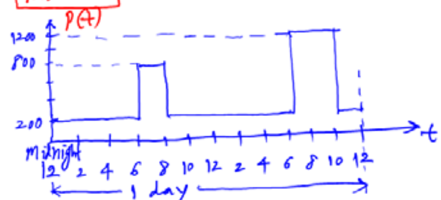
where currents i_1 and i_4 were assigned positive signs because they are labeled in the figure as entering the node, and i_2 and i_3 were assigned negative signs because they are leaving the node.

$i_1 - i_2 = 0$

$i_1 = i_2$

► Alternatively, the sum of currents leaving a node is zero, in which case we assign a "+" to a current leaving the node and a "−" to a current entering it. ◄

Prob.



a) Total energy consumed in one day in KWh?

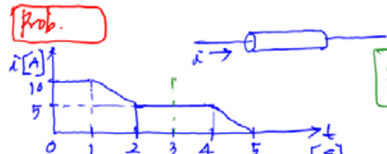
$200 \times 6 + 800 \times 2 + 200 \times 10 + 1200 \times 4 + 200 \times 2$

$= 1200 + 1600 + 2000 + 4800 + 400 = 10{,}000$ Wh $= 10$ KWh

b) Average power over the 24 hour period (day)?

$P_{avg} = \frac{1}{T}\int_0^T p(\tau)\,d\tau = \frac{10 \text{ KWh}}{24 \text{ h}} \approx 0.416$ KW

Prob.



$q(t) = \int_{-\infty}^{t} i(\tau)\,d\tau$

$q(0) = 0$

Find the total charge that passed through the element:

(a) $t = 1$ s, i.e. $q(t=1)$

$0 + \int_0^1 i(\tau)\,d\tau = 10 \text{ A} \cdot 1 \text{ s} = 10$ Coulomb [C]

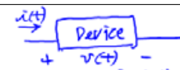
(b) $t = 3$ s, i.e. $q(t=3)$

$0 + \int_0^{t=3} i(\tau)\,d\tau = 10 + 7.5 + 5 = 22.5$ C

(c) $t = 5$ s, i.e. $q(t=5)$

$0 + \int_0^{t=5} i(\tau)\,d\tau = 22.5 + 7.5 = 30$ C

Prob.



For $t \geq 0$, $v(t) = 10\cos 2t$ [V], $i(t) = 20(1 - e^{-0.5t})$ [mA]

a) Find the charge in the device at $t = 2$ s $\quad q(0) = 0$

(assume initial charge at $t = 0$ is zero)

$e = 2.718 \quad e^{-1} = 0.368$

$0 + \int_0^{t=2} i(\tau)\,d\tau = \int_0^2 20(1 - e^{-0.5\tau})\,d\tau \quad$ [mA · s]

$= 20\left[\tau - \frac{e^{-0.5\tau}}{-0.5}\right]_0^2 = 20\left[2 - \frac{e^{-1} - 1}{-0.5}\right]$

$= 20[2 + (0.368 - 1) \times 2] = 20(2 - 1.264) = 14.76$ mC

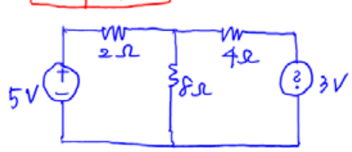
b) The power consumed by the device at $t = \pi$ s

$(e^{-\frac{\pi}{2}} \approx 0.208)$

$p(t) = v(t)\,i(t) = 10\underbrace{\cos(2\pi)}_{=1} \times 20\overbrace{(1 - \underbrace{e^{-0.5\pi}}_{0.208})}^{0.792}$

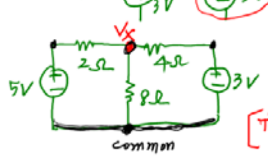
$= 200 \times 0.792 = 158.1$ V.mA $= 0.1584$ W

Example 1.10



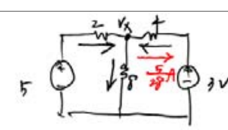
What is the polarity of 3V?

Case A $\quad$ 3V = 3V $\quad$ (Case B $\quad$ 3V = 3V)



Let's try to find V_x

[This is called Nodal Analysis]



KCL at ⓧ

$i_{2\Omega} + i_{4\Omega} = i_{8\Omega}$

$\frac{5 - V_x}{2} + \frac{3 - V_x}{4} = \frac{V_x}{8}$

$\times 8 \qquad 20 - 4V_x + 6 - 2V_x = V_x$

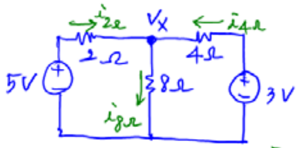
$20 + 6 = V_x + 4V_x + 2V_x$

$26 = 7V_x$

$V_x = \frac{26}{7}$ V

$i_{4\Omega} = \frac{3 - V_x}{4} = \frac{3 - \frac{26}{7}}{4} = \frac{21 - 26}{4 \times 7} = \frac{-5}{28}$ A

$V_x = 4\left(\frac{5}{28}\right) + 3 = \frac{5}{7} + 3 = \frac{26}{7}$ ✓



Current $i_{2\Omega}[A] = \dfrac{[5 - V_x][V]}{2[\Omega]}$ [Ohm's Law $v = R i$, $i = \dfrac{v}{R}$]

$i_{4\Omega}[A] = \dfrac{3 - V_x\,[V]}{4[\Omega]}$

$i_{8\Omega}[A] = \dfrac{V_x[V]}{8[\Omega]}$

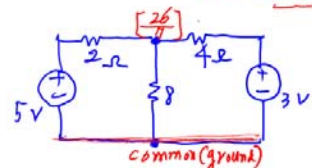
Also $i_{2\Omega} + i_{4\Omega} = i_{8\Omega}$ [Based on Kirchhoff's Current law $\sum_k i_k = 0$ at a node]

$$\frac{5-V_x}{2} + \frac{3-V_x}{4} = \frac{V_x}{8}$$

$$4(5-V_x) + 2(3-V_x) = V_x$$

$$20+6-4V_x-2V_x = V_x$$

$$26 = 7V_x \Rightarrow V_x = \frac{26}{7}\ [V]$$



$i_{2\Omega} = \dfrac{5-\frac{26}{7}}{2} = \dfrac{9}{14}\ [A]$

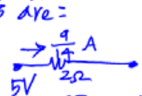
$i_{4\Omega} = \dfrac{3-\frac{26}{7}}{4} = -\dfrac{5}{28}\ [A]$ means in opposite direction

$i_{8\Omega} = \dfrac{\left(\frac{26}{7}\right)}{8} = \dfrac{13}{28}\ [A]$

$i_{8\Omega} = i_{2\Omega} + i_{4\Omega}$ (KCL ✓)

Also, Power absorbed in 2Ω, 4Ω, 8Ω resistors are:

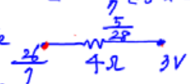
2Ω case



$$P_{2\Omega} = \left[\left(5-\frac{26}{7}\right)[V]\right]\left[\frac{9}{14}A\right]$$

$$= \frac{9}{7}[V] \times \frac{9}{14}[A] = \frac{81}{98}\ [W]$$

4Ω case



$$P_{4\Omega} = \left(\frac{26}{7}-3\right)[V] \times \frac{5}{28}[A] = \frac{5}{7}\times\frac{5}{28} = \frac{25}{196}\ [W]$$

8Ω case

$$P_{8\Omega} = \frac{26}{7}[V] \times \frac{13}{28}[A] = \frac{169}{98}\ [W]$$

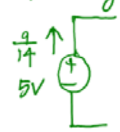
Total consumption

$$P_{total} = \frac{81}{98} + \frac{25}{196} + \frac{169}{98} = \frac{162+25+338}{196}$$

$$= \frac{525}{196} = \frac{75}{28}\ [W]$$

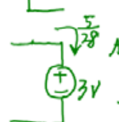
conservation (net zero)

Power generated by two voltage sources:



$$P_{5V} = -5[V] \times \frac{9}{14} = \frac{-45}{14}\ W$$

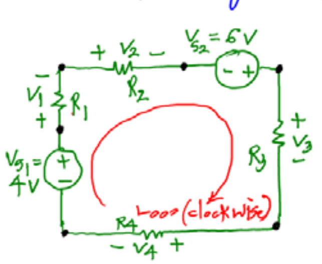
negative sign since this is supplied/generated

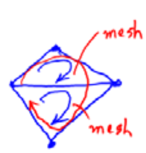


$$P_{3V} = +3[V] \times \frac{5}{28}[A] = +\frac{15}{28}\ [W]$$

Total generation $P_{5V} + P_{3V} = -\dfrac{45}{14} + \dfrac{15}{28} = -\dfrac{75}{28}\ [W]$

Kirchhoff's Voltage Law (KVL)





$$-4 + V_1 - V_2 - 6 + V_3 + V_4 = 0$$

According to KVL ($\sum_{\text{loop}} v_k = 0$) $\{s_1, 1, 2, s_2, 3, 4\}$

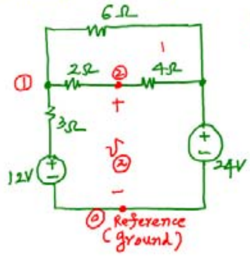
If we denote the current in the loop (loop current) as I, then (by Ohm's law $v_k = R_k I$)

$$-4 + R_1 I + R_2 I - 6 + R_3 I + R_4 I = 0$$

$$-(4+6) + (R_1+R_2+R_3+R_4) I = 0$$

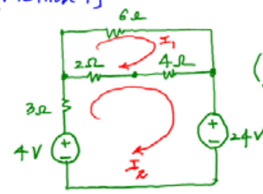
$$I = \frac{4+6}{R_1+R_2+R_3+R_4}$$

[Problem] from Fig 2-15 (a) on page 65



[Q] what is the voltage at ② ?

[Method 1] Solving by Loop Analysis – Loop Currents are unknown variables

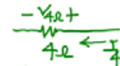


If we can find I_1, I_2, then

(i) the current (branch current) through 4Ω is

$$I_{4\Omega} = I_1 - I_2$$

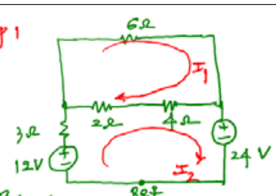
(ii)



$$V_{4\Omega} = 4\, I_{4\Omega} = 4\,(I_1 - I_2)$$

and (iii) $V_{②} = -V_{4\Omega} + 24$

Loop 1



By KVL in Loop 1

$$+6I_1 + 4(I_1 - I_2) + 2(I_1 - I_2) = 0$$

By KVL in Loop 2

$$-12 + 3I_2 + 2(I_2 - I_1) + 4(I_2 - I_1) + 24 = 0$$

$$\begin{bmatrix} 12 & -6 \\ -6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

row 1 × 3 + row 2 × 2 ⇒

$$\begin{array}{rcl} 36I_1 - 18I_2 &=& 0 \\ -12I_1 + 18I_2 &=& -24 \\ \hline 24I_1 + 0 &=& -24 \end{array} \Rightarrow I_1 = -1\,A$$

row 1 + row 2 × 2 ⇒

$$\begin{array}{rcl} 12I_1 - 6I_2 &=& 0 \\ -12I_1 + 18I_2 &=& -24 \\ \hline 0 + 12I_2 &=& -24 \end{array} \Rightarrow I_2 = -2\,A$$

Matrix solution

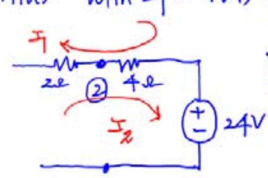
$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

$$\underset{\sim}{x} = \underset{\sim}{A}^{-1}\,\underset{\sim}{b}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -12 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 9 & 6 \\ 6 & 12 \end{bmatrix}}{\begin{vmatrix} 12 & -6 \\ -6 & 9 \end{vmatrix}} \begin{bmatrix} 0 \\ -12 \end{bmatrix} = \frac{\begin{bmatrix} -72 \\ -144 \end{bmatrix}}{12(9) - (-6)(-6) = 72} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

Thus with $I_1 = -1\,A$, $I_2 = -2\,A$

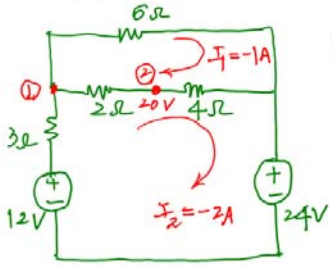


$$V_{②} = 4\Omega(I_2 - I_1) + 24$$

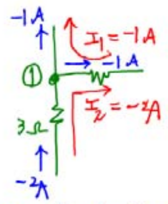
$$= 4(-2-(-1)) + 24$$

$$= 20\,V$$

Revisit the Circuit

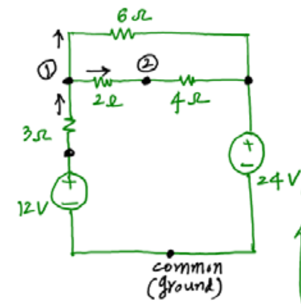


check KCL at ①



Leaving $-1 - 1 = -2A$
entering $-2A$ =

Method 2 : Nodal Analysis – node voltages are unknown variables



Only node voltages at ①, ② are unknown

$\Rightarrow \{V_1, V_2\}$ to solve for.

KCL at ①:

$$\frac{12-V_1}{3} = \frac{V_1-24}{6} + \frac{V_1-V_2}{2} \quad \text{(eq.1)}$$

KCL at ②:

$$\frac{V_1-V_2}{2} = \frac{V_2-24}{4} \quad \text{(eq.2)}$$

$6 \times$ eq(1) $\Rightarrow 24 - 2V_1 = V_1 - 24 + 3V_1 - 3V_2$

or $48 = 6V_1 - 3V_2$ (eq 1)'

$4 \times$ eq(2) $\Rightarrow 2V_1 - 2V_2 = V_2 - 24$

or $2V_1 - 3V_2 = -24$ (eq 2)'

In matrix form

$$\begin{bmatrix} 6 & -3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 48 \\ -24 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 24 \begin{bmatrix} 6 & -3 \\ 2 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 24 \frac{\begin{bmatrix} -3 & 3 \\ -2 & 6 \end{bmatrix}}{\begin{vmatrix} 6 & -3 \\ 2 & -3 \end{vmatrix}} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \frac{24}{-12} \begin{bmatrix} -9 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 20 \end{bmatrix}$$