In Lecture 1, we covered:
- Circuit elements R, L, C, sources, dependent sources
- Kirchhoff Current Law (KCL)
- Kirchhoff Voltage Law (KVL)
- Power generation $V_I < 0$
- Power consumption $V_I > 0$
- Conservation of power $\sum P_k = 0$

2.1 - 2.2 Characteristics of Ideal Resistor

Based on the results of his experiments on the nature of conduction in silicate, German Physicist Georg Simon Ohm (1789-1854) formulated in 1827 the $i \propto v$ relationship for a resistor, which has become known as Ohm's law. He discovered that the voltage across a resistor is directly proportional to the current $i$ flowing through it, namely

$$v = ir$$

with the resistance $R$ being the proportionality factor.

Figure 2.6: $i \propto v$ relationship for an ideal resistor.

An ideal linear resistor is one whose resistance $R$ is constant and independent of the magnitude of the current flowing through it, i.e., it obeys Ohm’s law (Fig. 2.6).
A common convention is to assign a positive "+" sign to a current if it is entering the node and a negative "-" sign if it is leaving it.

For the node in Fig. 3-9, the sum of currents entering the node is
\[ i_1 - i_2 - i_3 + i_4 = 0, \]
where currents \( i_1 \) and \( i_2 \) were assigned positive signs because they are labeled as entering the node, and \( i_3 \) and \( i_4 \) were assigned negative signs because they are leaving the node.

Alternatively, the sum of currents leaving a node is zero, in which case we assign a "-" to a current leaving the node and a "+" to a current entering it.

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**Example 1.10**

Find the total charge that passed through the elements:

(a) \[ \int_{t_1}^{t_2} i(t) \, dt = 10 \] Coulomb

(b) \[ \int_{t_1}^{t_2} 2i(t) \, dt = 20 \] Coulomb

(c) \[ \int_{t_1}^{t_2} 5i(t) \, dt = 50 \] Coulomb

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**Problem 1.2**

Find the power consumed in one day in kw-hr.

\[ \text{Total energy consumed in one day} = \text{kw-hr} \]
\[ = \frac{4 \times 6 + 8 \times 2 + 2 \times 10 + 12 \times 9 + 6 \times 2}{10} = 10 \text{kw-hr} \]

1. Average power over the 24 hour period (day) is
\[ P_{avg} = \frac{1}{24} \int_{0}^{24} P(t) \, dt = \frac{10 \times 24}{24} = 10 \text{ kw} \]

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**Problem 2.3**

Calculate the power of the device at \( t = 3 \) sec.

\[ P(t) = 5(t - 1)^2 \text{watts} \]
\[ P(3) = 5(3 - 1)^2 = 5 \times 4 = 20 \text{watts} \]

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**Problem 3.4**

Calculate the voltage across the capacitor at \( t = 2 \) sec.

\[ V(t) = 10 \times (1 - e^{-t/10}) \text{volts} \]
\[ V(2) = 10 \times (1 - e^{-2/10}) \approx 9.54 \text{ volts} \]

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**Example 3.5**

What is the polarity of the 3V source?

**Case A:** \[ V_1 = +3V \]

**Case B:** \[ V_2 = -3V \]

Let's try to find \( V_3 \)

This is called Nodal Analysis.
Current: $\frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2}$

According to KVL, $-4 + V_1 - V_2 = 6 + V_2 + V_3 = 0$

Also, $\frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2} = \frac{\frac{5}{2} - \frac{3}{2}V_x}{2}$

Total consumption: $P_{total} = \frac{P_5V}{14} + \frac{\frac{1}{4}}{4} + \frac{\frac{1}{4}}{4} = \frac{P_5V}{14} + \frac{\frac{1}{4}}{4} + \frac{\frac{1}{4}}{4}

Power generated by two voltage sources:

If we denote the current in the loop (1st current branch) as $I$, then (by Ohm's law $V_e - 5 = I$)

$-4 + I + R_1 - V_e - 5 + R_2 + R_3 - 5 = 0$

$-I + R_1 + R_2 + R_3 - 10 = 0$

$I = \frac{V_e}{R_1 + R_2 + R_3}$
Problem from Fig. 2-15 (a) on page 65

**What is the voltage at (?)**

**Method 1**

Solved by loop analysis - Loop currents are unknown variables

1. We can find \( I_1, I_2 \) if we first know the current (branch current) through \( 4 \Omega \) is

\[
\begin{align*}
I_a &= I_1 - I_2 \\
V_2 &= 4V - I_a
\end{align*}
\]

2. Therefore

\[
I_a = 4 \Rightarrow \frac{V_2}{4} = 4 - \frac{4}{4} = 4 - 1 = 3
\]

And

\[
V_2 = -3.5 + 3V_2 = 4
\]

**Loop 1**

\[
\begin{align*}
12 V &= 6I_1 + 4(I_1 - I_2) + 2(I_1 - I_1) = 0 \\
12V &= 6I_1 - 6I_2 + 8I_1 - 2I_2 + 12 = 0
\end{align*}
\]

\[
\begin{bmatrix}
12 & -6 \\
-6 & 9
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
0 \\
-12
\end{bmatrix}
\]

Matrix solution

\[
\begin{bmatrix}
A \\
\bar{b}
\end{bmatrix} \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = \begin{bmatrix}
\bar{b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = \begin{bmatrix}
A
\end{bmatrix}^{-1} \begin{bmatrix}
\bar{b}
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \begin{bmatrix}
12 & -6 \\
-6 & 9
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
-12
\end{bmatrix}
\]

Thus with \( I_1 = -1A, I_2 = -2A \)

\[
\begin{align*}
\begin{bmatrix}
24V \\
V_2
\end{bmatrix} &= 4 \begin{bmatrix}
I_2 \\
I_1
\end{bmatrix} + 24 \\
&= 4(-2) + (-1) + 24 \\
&= 20V
\end{align*}
\]
**Revisit the Circuit**

Check KCL at (1):

Leaving -1 -1 -1 A

Entries: -3 A =

6 × Eq.(1) ⇒ 24 - 2V₁ = V₁ - 24 + 3V₁ - 3V₂
t or 4V = 6V₁ - 8V₂ (eq.1)

4 × Eq.(3) ⇒ 2V₁ - 2V₂ = V₂ - 24
t or 2V₁ - 3V₂ = -24 (eq.2)

In matrix form:

\[
\begin{bmatrix}
6 & -9 \\
2 & -3
\end{bmatrix}
\begin{bmatrix}
V₁ \\
V₂
\end{bmatrix} =
\begin{bmatrix}
4P \\
24
\end{bmatrix}
\]

\[
V₂ = 24 \left[ \begin{array}{cc}
6 & -9 \\
2 & -3
\end{array} \right]^{-1} \left[ \begin{array}{c}
4P \\
24
\end{array} \right]
= 24 \left[ \begin{array}{cc}
\frac{2}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{2}{3}
\end{array} \right] \left[ \begin{array}{c}
4P \\
-24
\end{array} \right]
= \left[ \begin{array}{c}
18P \\
-24
\end{array} \right]
\]

**Method 2: Node Analysis - node voltages are unknown variables**

Only node voltages at (1), (2) are unknown
\(\{V₁, V₂\}\) to solve

KCL at (1):

\[
\frac{12-V₁}{3} = \frac{V₁-24}{6} \quad \text{(eq.1)}
\]

KCL at (2):

\[
\frac{V₁-V₂}{2} = \frac{V₂-24}{4} \quad \text{(eq.2)}
\]

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