EE101 W19 Lecture 18, May 7, 2019
HW #9 for Quiz 9 on May 9

[1] 6.29
[2] 6.35
[3] 6.38
[4] 6.50
[5] 6.54

Average = 6.31

\[ \Delta t = \Delta f \]
\[ \frac{d}{dt} = j\omega \]

Table 7-4: Time-domain sinusoidal functions \( x(t) \) and their complex-frequency counterparts \( X(j\omega) \), where
\[ x(t) = \mathcal{F}[x(t)] \]

<table>
<thead>
<tr>
<th>Property</th>
<th>( \mathcal{F} )</th>
<th>( L )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( \mathcal{F}(t) )</td>
<td>( \frac{1}{s} )</td>
<td>( \frac{1}{s} )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( \delta(t) )</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Phase domain circuit analysis

Step 1: \( A = \mathcal{F}(x(t)) \)
Step 2: \( \mathcal{F} \) to \( j\omega \) domain
Step 3: Write the KVL equation
Step 4: Solve for \( x(t) \)
Solution: (a) We start by converting \( v_h(t) \) to cosine format:

\[
v_h(t) = 4 \sin(10^7 t + 15^\circ)
\]

\[
= 4 \cos(10^7 t + 15^\circ - 90^\circ) = 4 \cos(10^7 t - 75^\circ) \ V.
\]

The corresponding phasor voltage is

\[
V_h = 4 e^{-j75^\circ} \ V,
\]

and the impedances shown in Fig. 7-13(b) are given by

\[
Z_{R_1} = R_1 = 10 \ \Omega,
\]

\[
Z_C = \frac{-j}{\omega C} = \frac{-j}{10^7 \times 10^{-8}} = -j10 \ \Omega,
\]

(b) The current is given by

\[
I = \frac{V_h}{Z_{R_1} + Z_C} = \frac{4 e^{-j75^\circ}}{10 + 10} = 0.2 e^{-j75^\circ} A.
\]

The corresponding current in the time domain is

\[
I(t) = 0.2 \cos(10^7 t - 75^\circ) + 0.2 \sin(10^7 t - 75^\circ) \ A.
\]
\[ \Delta \rightarrow Y \text{ transformation:} \]

\[ Z_1 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}, \quad (7.79a) \]

\[ Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}, \quad (7.79b) \]

\[ Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}. \quad (7.79c) \]

\[ Z_1 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} = \frac{-j6 \times 12}{24 - j12 - j6 + 12} = \frac{-j72}{36 - j18} = (0.8 - j1.6) \, \Omega, \]

\[ Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} = \frac{(24 - j12) \times 12}{36 - j18} = 8 \, \Omega, \]

and

\[ Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} = \frac{-j6(24 - j12)}{36 - j18} = -j4 \, \Omega. \]

In Fig. 7-18(c), \( Z_d \) represents the series combination of \( Z_3 \) and \( Z_d \).
Open-circuit / short-circuit method

\[ Z_{TH} = \frac{V_{OC}}{I_{SC}}, \hspace{1cm} (7.53) \]

where \( I_{SC} \) is the short-circuit current at the circuit’s output terminals (Fig. 7.18(a)).

\[ V_{TH} = V_{S} \frac{Z_{S}}{Z_{S} + Z_{Z}} \]
\[ Z_{TH} = Z_{S} \parallel Z_{Z} \]

\[ V_{TH} = V_{S} \frac{Z_{S} \parallel Z_{Z}}{Z_{S} + Z_{Z}} = V_{S} \frac{2 - \frac{j\pi}{2}}{\pi} \]

\[ = (7.08 + j2.10) \]
\[ = 7.08 + j4.92 \]
\[
A = \begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}, \quad A^{-1} = \frac{1}{|A|} \begin{bmatrix}
  d & -b \\
  -c & a
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
  2 & 4 \\
  1 & 3
\end{bmatrix}, \quad B^{-1} = \frac{1}{2} \begin{bmatrix}
  3 & -4 \\
  1 & 2
\end{bmatrix}
\]

\[
|B| = 6 - 4 = 2
\]

\[
|A| = 2 + 3 = 5
\]
\[
\begin{bmatrix}
1 & 5 & -4 \\
-1 & -3 & 6 \\
1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix} =
\begin{bmatrix}
10 \\
0 \\
5
\end{bmatrix}. \\
(\text{B.4})
\]

Note that \(a_{11} = 1\), \(a_{21} = -1\), and \(a_{31} = 0\). The regularized set of three linear, simultaneous equations given by Eq. (B.4) is a system of order 3.

**Step 2: General Solution**

According to Cramer’s rule, the solutions for \(i_1\) to \(i_3\) are given by

\[
i_1 = \frac{\Delta_1}{\Delta}, \quad (\text{B.5a})
\]

\[
i_2 = \frac{\Delta_2}{\Delta}, \quad (\text{B.5b})
\]

\[
i_3 = \frac{\Delta_3}{\Delta}, \quad (\text{B.5c})
\]

Each cofactor is a \(2 \times 2\) determinant. Application of the definition given by Eq. (B.9) leads to

\[
\text{adj } A = \begin{bmatrix}
6 & 4 & 18 \\
4 & 6 & -2 \\
4 & 6 & 2
\end{bmatrix}. \quad (\text{B.27})
\]

Upon incorporating Eqs. (B.22) and (B.23) and using the value of \(\Delta\) obtained in Eq. (B.13), we have

\[
\text{I} - \begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix} = \frac{1}{20} \begin{bmatrix}
6 & 4 & 18 \\
4 & 6 & -2 \\
4 & 6 & 2
\end{bmatrix} = \begin{bmatrix}
10 \\
0 \\
5
\end{bmatrix}. \quad (\text{B.28})
\]

Standard matrix multiplication leads to

\[
i_1 = \frac{1}{20} \begin{bmatrix}
6 & 4 & 18 \\
4 & 6 & -2 \\
4 & 6 & 2
\end{bmatrix} \begin{bmatrix}
10 \\
0 \\
5
\end{bmatrix} = \frac{1}{20}(6 \times 10 + 4 \times 0 + 18 \times 5) = 7.5. \quad (\text{B.29})
\]

Similarly, multiplication using the second and third rows of \(\text{adj } A\) leads to \(i_2 = i_3 = 2.5\).

**Creating the Adjugate Matrix to Find the Inverse Matrix**

\[
M = \begin{bmatrix}
1 & 2 & 3 \\
0 & 1 & 4 \\
5 & 6 & 0
\end{bmatrix}
\]

\[
\text{det}(M) = 1(0-24) - 2(0-20) + 3(0-5) = 1
\]

\[
M^T = \begin{bmatrix}
1 & 2 & 3 \\
0 & 1 & 4 \\
5 & 6 & 0
\end{bmatrix}^T
\]

\[
M^T = \begin{bmatrix}
1 & 0 & 5 \\
2 & 1 & 6 \\
3 & 4 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
16 \\
40 \\
05
\end{bmatrix} = -24
\]

\[
\begin{bmatrix}
26 \\
30 \\
15
\end{bmatrix} = -18
\]

\[
\begin{bmatrix}
21 \\
34 \\
10
\end{bmatrix} = 5
\]

\[
\begin{bmatrix}
-20 \\
-15 \\
-16
\end{bmatrix} = 4
\]

\[
\begin{bmatrix}
10 \\
21 \\
1
\end{bmatrix} = 1
\]
Example 7.12: Nodal Analysis

Apply the nodal-analysis method to determine $i_1(t)$ in the circuit of Fig. 7.25(a). The sources are given by:

- $u_{C1}(t) = 12 \cos 10^3 t \mathrm{V}$
- $u_{C2}(t) = 6 \sin 10^3 t \mathrm{V}$

**Solution:** We first demonstrate how to solve this problem using the standard nodal-analysis method (Section 3.2), and then we solve it again by applying the by-inspection method (Section 3.6).

Nodal-analysis method

Our first step is to transform the given circuit to the phasor domain. Accordingly,

\[
Z_C = \frac{1}{\omega C} = \frac{-j}{10^3 \times 0.25 \times 10^{-3}} = -j 4 \Omega,
\]

\[
Z_L = \omega L = j 10^3 \times 10^{-3} = j 1 \Omega,
\]

\[
u_C(t) = 12 \cos 10^3 t \quad \rightarrow \quad V_C = 12 \mathrm{V},
\]

and

\[
u_L(t) = 6 \sin 10^3 t \quad \rightarrow \quad V_L = -6 \mathrm{V},
\]

where for $V_C$, we used the property given in Table 7-2, namely that the phasor counterpart of $\sin \omega t$ is $-j$. Using these values, we generate the phasor-domain circuit given in Fig. 7.25(b) in
11.2 Transformers

11.2.1 Coupling Coefficient

To couple magnetic flux between two coils, the coils may be wound around a common core (Fig. 11.8) so that the relative magnitudes of the magnetic flux linkage of the primary and secondary windings are such that the coupling coefficient is well defined. A typical coupling coefficient (a.k.a. coefficient of coupling) can be defined as

\[ k = \frac{N_1 N_2 M}{\sqrt{N_1^2 L_1 + N_2^2 L_2}} \]

where:
- \( k \) is the coupling coefficient.
- \( N_1 \) and \( N_2 \) are the number of turns in the primary and secondary windings, respectively.
- \( M \) is the mutual inductance of the windings.
- \( L_1 \) and \( L_2 \) are the self-inductances of the primary and secondary windings, respectively.

Most core materials, including air, wood, and ceramic, are non-magnetic, permitting a maximum of magnetic flux density. The magnetic field intensity is inversely proportional to the reluctance of the magnetic path. Thus, the material with the least reluctance will accumulate the greatest flux density. Various types of magnetic materials can be used to maximize the magnetic field intensity. For example, iron is a commonly used material due to its high magnetic permeability.

Figure 11.9: Electromagnetic coupling.

The transformer may be visualized as a matrix form:

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} N_1 & M \\ M & N_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]

where:
- \( V_1 \) and \( V_2 \) are the primary and secondary voltages, respectively.
- \( I_1 \) and \( I_2 \) are the primary and secondary currents, respectively.

The transformer and its equivalent circuit exhibit the same V-I relationships of the windings in the circuit of Eq. (11.27).

Figure 11.10: Transformer circuit.

7.12 Ideal Transformers

A transformer consists of two windings called windings, that are in close proximity to each other but not connected electrically. The two windings are called the primary and the secondary, as shown in Fig. 7.36. Even though the two windings are isolated electrically—meaning that no current flows between them—when an alternating voltage is applied to the primary, it creates a magnetic flux that permeates both windings through a common area, inducing an ac voltage in the secondary.

The transformer gets its name from the fact that it is used to transform currents, voltages, and impedances between its primary and secondary circuits.

The key parameter that determines the relationships between the primary and the secondary is the turns ratio \( n = N_2 / N_1 \).

Data on same ends

\[ L_2 = L_1 - M \]  
\[ L_2 = L_2 - M \]  
\[ L_2 = M \]  

Data on opposite ends

\[ L_1 = L_1 + M \]  
\[ L_2 = L_2 + M \]  
\[ L_2 = -M \]  

Even though a negative value for inductance \( L_2 \) is not physically realizable, the mathematical equivalence holds nonetheless and the equivalent circuit is perfectly applicable.

Figure 7.36: Schematic symbol for an ideal transformer.
where \( N_1 \) is the number of turns in the primary coil and \( N_2 \) is the number of turns in the secondary. As an additional note, it's important to consider the direction of the voltage, which is relative to the direction of the secondary winding, relative to the direction of the primary winding. In general, the voltage across the secondary is given by:

\[
E_2 = N_2 E_1
\]  

where \( N_1 \) and \( N_2 \) are defined such that the voltage across the secondary coil is in the same direction as the voltage across the primary coil when the power is positive. If all the power supplied by a source to its primary coil is transferred to the load connected at the secondary side, then \( \phi = \phi_2 \) and since \( N_1 = N_2 \), it follows that:

\[
\frac{E_1}{E_2} = \frac{N_1}{N_2}
\]

This is always defined as the direction towards the axis on the primary side and 1 is defined as the direction away from the axis on the secondary side. The purpose of the sign designation is to indicate whether the windings in the primary and secondary coils are in the same (clockwise) or opposite (counter-clockwise) direction.

Example 7-5: Filter Design

If the bridge rectifier circuit of Fig. 7-23 has an input sine wave signal, determine the values of \( R \) and \( C \) that would result in the waveform shown in Fig. 7-24, where \( T_{on} / T_{off} \) is the period of the sine wave. Assume \( T_{on} = T_{off} = T \).

Solution: If the frequency of the original sinusoidal input is \( f \), the frequency of the output waveform is \( 2f \) since the process is taking place over \( 180^\circ \). Thus, the period of the rectified waveform is

\[
T_{on} = \frac{T}{2}
\]

and the corresponding design specifications are:

\[
\frac{T_{on}}{T_{off}} = 0.60 \text{ sec} \quad \text{and} \quad \frac{T_{on}}{T} = 0.10 \text{ sec}
\]

Application of Eq. (7.145) leads to

\[
T_{on} = 0.25 T
\]

Figure 7-24: Bridge rectifier circuit.
discussed in Chapter 6. \( Y_{O} / Y_{I} = 4 \), \( C = 2 \mu F \), \( R_{e} = 50 \Omega \),
\( R_{i} = 140 \Omega \), \( R_1 = 39 \Omega \), \( V_{o} = 200 \text{V} \), and \( V_{i} = 30 \text{V} \). Electronic \( \eta \), the
double voltage, and the double-frequency relative to \( v_{o} \).

Solution: At the secondary side of the transformer,

\[
\eta = \frac{V_o}{V_i} \approx \frac{200}{30} \approx 6.67 V.
\]

Hence, \( V_o = 33.1 \text{V} \), which is greater than the input voltage
\( V_i = 30 \text{V} \).

Consequently, the output will not be the input voltage.

In Example 7.39, we established that \( V_o / V_i = 0.35 \text{mV} \). Also,
\( R_i = 100 \Omega \), \( R_1 = 100 \Omega \), \( C = 0.35 \mu F \), \( V_1 = 30V \).

Application of Eq. (7.147) gives

\[
\begin{align*}
\eta &= \frac{V_o}{V_i} = \frac{200}{30} \\
&= 6.67 \\
&
\end{align*}
\]

Hence,

\[
\frac{v_o}{v_i} = 0.35 \text{V} / \text{V} = 0.0028.
\]

which represents a relative variation of less than \(0.2\) percent.