EE 101 Lecture 15, Feb 26, 2019

Quiz 8 on March 4 based on HW 8.

[1] Prob 6.1
[2] 6.3
[3] 6.7
[4] 6.12
[5] 6.16
[6] 6.18
[7] 6.22
[8] 6.25

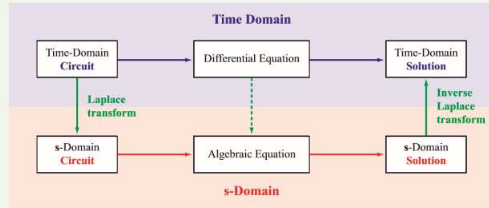
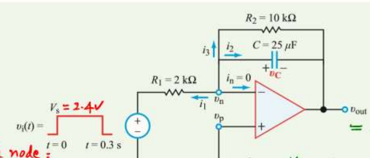


Figure 12-2: The top horizontal sequence involves solving a differential equation entirely in the time domain. The bottom horizontal sequence involves a much easier solution of a linear equation in the **s**-domain.

Table 12-1: Properties of the Laplace transform ($f(t) = 0$ for $t < 0^-$).

Property	$f(t)$		$\mathbf{F}(\mathbf{s}) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K\ f(t)$	$\leftrightarrow$	$K\ \mathbf{F}(\mathbf{s})$
2. Linearity	$K_1\ f_1(t) + K_2\ f_2(t)$	$\leftrightarrow$	$K_1\ \mathbf{F}_1(\mathbf{s}) + K_2\ \mathbf{F}_2(\mathbf{s})$
3. Time scaling	$f(at), \quad a > 0$	$\leftrightarrow$	$\frac{1}{a}\ \mathbf{F}\left(\frac{\mathbf{s}}{a}\right)$
4. Time shift	$f(t-T)\ u(t-T)$	$\leftrightarrow$	$e^{-T\mathbf{s}}\ \mathbf{F}(\mathbf{s}), \quad T \ge 0$
5. Frequency shift	$e^{-at}\ f(t)$	$\leftrightarrow$	$\mathbf{F}(\mathbf{s}+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	$\leftrightarrow$	$\mathbf{s}\ \mathbf{F}(\mathbf{s}) - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2f}{dt^2}$	$\leftrightarrow$	$\mathbf{s}^2\mathbf{F}(\mathbf{s}) - \mathbf{s}\ f(0^-) - f'(0^-)$
8. Time integral	$\int_{0^-}^{t} f(\tau)\ d\tau$	$\leftrightarrow$	$\frac{1}{\mathbf{s}}\ \mathbf{F}(\mathbf{s})$
9. Frequency derivative	$t\ f(t)$	$\leftrightarrow$	$-\frac{d}{d\mathbf{s}}\ \mathbf{F}(\mathbf{s})$
10. Frequency integral	$\frac{f(t)}{t}$	$\leftrightarrow$	$\int_{\mathbf{s}}^{\infty} \mathbf{F}(\mathbf{s}')\ d\mathbf{s}'$



(a) Op-amp circuit

$= -v_C$

KCL at v_n node:

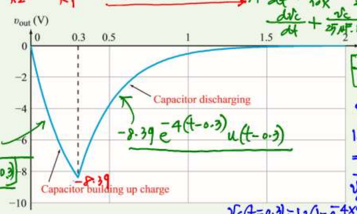
$$C\frac{dv_c}{dt} + \frac{v_c}{R_2} = \frac{v_i}{R_1}$$

For $0 < t < 0.3$

$$25\mu F\frac{dv_c}{dt} + \frac{v_c}{10K} = \frac{2.4}{2K}$$

$$\frac{dv_c}{dt} + \frac{v_c}{25\mu F \cdot 10K} = \frac{2.4}{25\mu F \cdot 2K}$$

$$\boxed{\frac{dv_c}{dt} + 4v_c = 48}$$



(b) $v_{out}(t)$

$$\underline{-12(1-e^{-4t}) \times [u(t) - u(t-0.3)]}$$

$$\underline{-8.39\ e^{-4(t-0.3)}\ u(t-0.3)}$$

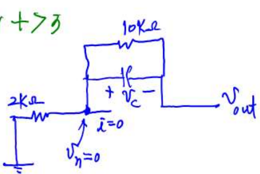
-8.39

$v_c(t) =$
$12 + [0-12]\ e^{-4t}$
$= 12(1 - e^{-4t})$

$v_o = -v_c(t)$

$$v_c(t=0.3) = 12(1-e^{-4\times 0.3}) = 12(0.699) = 8.39$$

For $t > 3$



$$v_o = -v_c$$

$$25\mu F\frac{dv_c}{dt} = -\frac{v_c}{10K\Omega}$$

$$\frac{dv_c}{dt} + \frac{1}{25\mu F\ 10K\Omega}v_c = 0$$

$$v_c(3) = 8.39$$

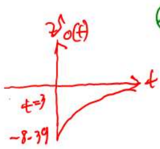
$$s V_c(s) - 8.39 + 4\ V_c(s) = 0$$

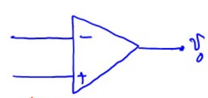
$$(s+4)\ V_c(s) = 8.39$$

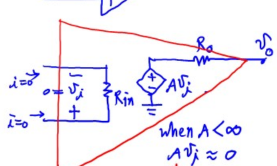
$$V_c(s) = \frac{8.39}{s+4}$$

$$\underline{v_c(t) = 8.39\ e^{-4(t-3)}\ u(t-3)}$$

$$v_o(t) = -v_c(t) = -8.39\ e^{-4(t-3)}\ u(t-3)$$

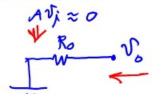






when $A < \infty$

$Av_i \approx 0$



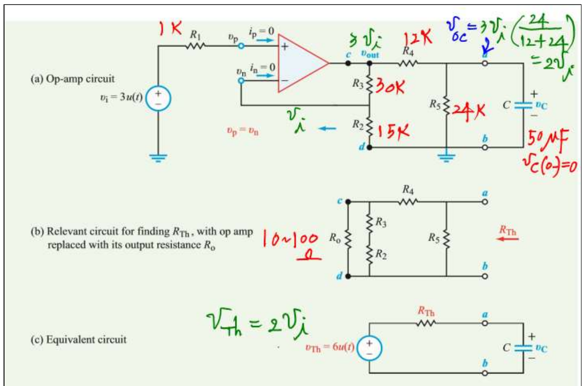
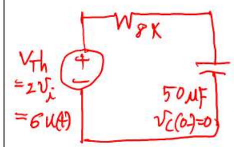


Figure 5-44: Circuit for Example 5-17.

$$R_{TH} = \left[\underbrace{\{\underbrace{R_0 \| (R_2+R_3)}_{\text{small}}\} + R_4}_{\approx R_4}\right] \| R_5$$

$$\simeq R_4 \| R_5 = \frac{12K \cdot 24K}{12K + 24K} = \frac{12K \cdot 24K}{36K} = 8\ K\Omega$$

$$6u(t) = 8K\left[50\mu F \frac{dv_C(t)}{dt}\right] + v_C(t)$$



$$\frac{dv_C(t)}{dt} + \frac{v_C}{8K \cdot 50\mu} = \frac{6}{8K \cdot 50\mu}$$

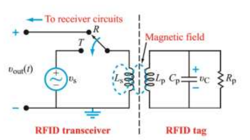
$$\frac{dv_C}{dt} + 2.5\, v_C = 15$$

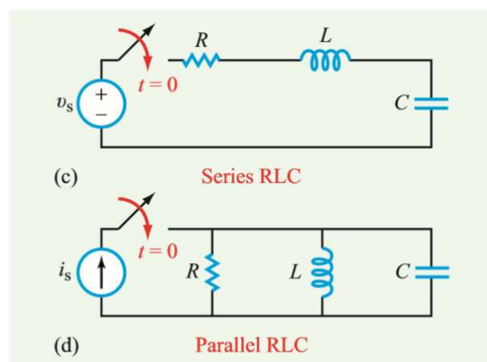
$$v_C(\infty) = 6,\ v_C(0) = 0,\ a = 2.5$$

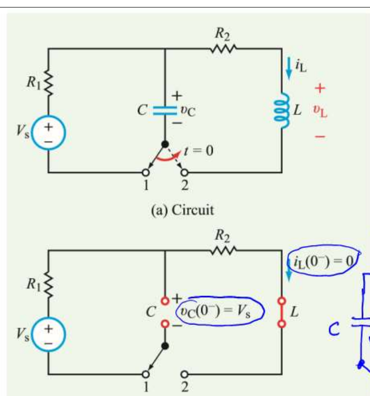
$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\, e^{-at},\ t > 0 \Rightarrow v_C(t) = 6\left[1 - e^{-2.5t}\right] \times u(t)$$

RLC Circuits

Contents

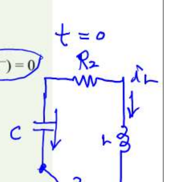




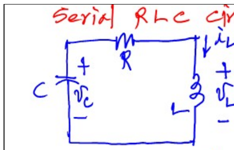


(a) Circuit

(b) At $t = 0^-$, C acts like an open circuit and L like a short circuit



Serial RLC circuit



$i_L(0^-) = i_L(0) = 0$

In order to find $i_L'(0)$ and $v_C'(0)$, consider

$v_C(0^-) = v_C(0) = V_s$

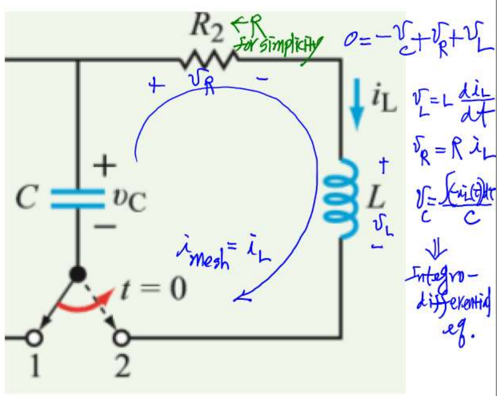
(i) $L\frac{di_L(t)}{dt} = v_C - R\, i_L$

At $t=0$ $\quad L\frac{di_L(0)}{dt} = \underbrace{v_C(0)}_{V_s} - R\, \underbrace{i_L(0)}_{0}$

$$\Rightarrow \boxed{i_L'(0) = \frac{V_s}{L}} \quad (1)$$

(ii) $C\frac{dv_C(0)}{dt} = -i_L(0)$

$$\boxed{v_C'(0) = -\frac{i_L(0)}{C} = 0} \quad (2)$$



$$\Rightarrow +\frac{1}{C}\int_{-\infty}^{t} i_L(\tau)\,d\tau + R\,i_L(t) + L\frac{di_L(t)}{dt} = 0$$

$$\frac{d}{dt}$$

$$-\frac{1}{C}\,i_L(t) + R\frac{di_L(t)}{dt} + L\frac{d^2 i_L(t)}{dt^2} = 0$$

dividing by L

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L(t)}{dt} + \frac{1}{LC}\,i_L(t) = 0} \quad (1)$$

Table 12-1: Properties of the Laplace transform ($f(t) = 0$ for $t < 0^-$).

Property	$f(t)$		$\mathbf{F(s)} = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K\,f(t)$	↔	$K\,\mathbf{F(s)}$
2. Linearity	$K_1\,f_1(t) + K_2\,f_2(t)$	↔	$K_1\,\mathbf{F_1(s)} + K_2\,\mathbf{F_2(s)}$
3. Time scaling	$f(at),\quad a > 0$	↔	$\frac{1}{a}\mathbf{F}\left(\frac{\mathbf{s}}{a}\right)$
4. Time shift	$f(t-T)\,u(t-T)$	↔	$e^{-T\mathbf{s}}\,\mathbf{F(s)},\quad T \geq 0$
5. Frequency shift	$e^{-at}\,f(t)$	↔	$\mathbf{F}(\mathbf{s}+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	↔	$\mathbf{s}\,\mathbf{F(s)} - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2 f}{dt^2}$	↔	$\mathbf{s}^2\mathbf{F(s)} - \mathbf{s}\,f(0^-) - f'(0^-)$
8. Time integral	$\int_{0^-}^{t} f(\tau)\,d\tau$	↔	$\frac{1}{\mathbf{s}}\,\mathbf{F(s)}$
9. Frequency derivative	$t\,f(t)$	↔	$-\frac{d}{d\mathbf{s}}\,\mathbf{F(s)}$
10. Frequency integral	$\frac{f(t)}{t}$	↔	$\int_{\mathbf{s}}^{\infty} \mathbf{F}(\mathbf{s}')\,d\mathbf{s}'$

$$s^2 I_L(s) - s\,i_L(0) - i_L'(0) + \frac{R}{L}\left(s\,I_L(s) - i_L(0)\right) + \frac{1}{LC}I_L(s) = 0$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)I_L(s) = \left(s + \frac{R}{L}\right)i_L(0) + i_L'(0)$$

Previously we found $i_L(0) = 0$

$i_L'(0) = \frac{V_S}{L}$

$$\Rightarrow \boxed{I_L(s) = \frac{\frac{V_S}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\frac{R}{2L}s + \omega_0^2, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \left(s + \underbrace{\frac{R}{2L}}_{=\alpha}\right)^2 + \omega_0^2 - \left(\frac{R}{2L}\right)^2$$

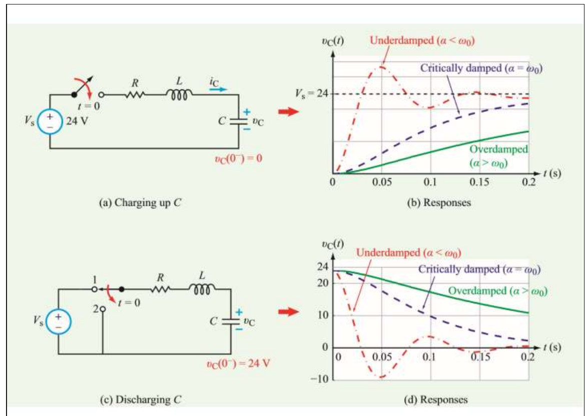
If $\omega_0 = \frac{R}{2L} (= \alpha)$, critically damped

If $\omega_0 > \frac{R}{2L} (= \alpha)$, underdamped

If $\omega_0 < \frac{R}{2L} (= \alpha)$, overdamped

where,

$\alpha = \frac{R}{2L}$ = damping coefficient, $\omega_0 = \frac{1}{\sqrt{LC}}$ = resonant freq.



Underdamped Case ($\omega_0 > \frac{R}{2L} = \alpha$)

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\alpha s + \omega_0^2 = (s+\alpha)^2 + \omega_0^2 - \alpha^2$$

$$= (s+\alpha)^2 - (j\underbrace{\sqrt{\omega_0^2-\alpha^2}}_{=\beta})^2 = (s+\alpha+j\beta)(s+\alpha-j\beta)$$

$$\Rightarrow I_L(s) = \frac{V_s}{L}\,\frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)}$$

$$= \frac{V_s}{L}\left[\frac{A_1}{s+\alpha+j\beta} + \frac{A_2}{s+\alpha-j\beta}\right]$$

$$A_1 = \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)}\times(s+\alpha+j\beta)\Big|_{s+\alpha+j\beta=0}$$

$$= \frac{1}{-\alpha-j\beta+\alpha-j\beta} = \frac{1}{-j2\beta}$$

$$A_2 = \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)}\times(s+\alpha-j\beta)\Big|_{s+\alpha-j\beta=0} = \frac{1}{+j2\beta}$$

$$\Rightarrow I_L(s) = \frac{V_s}{L}\left[\frac{\frac{1}{-j2\beta}}{s+\alpha+j\beta} + \frac{+\frac{1}{j2\beta}}{s+\alpha-j\beta}\right]$$

$\downarrow \mathcal{L}^{-1}$

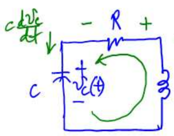
$$i_L(t) = \frac{V_s}{L}\,\frac{1}{\beta}\left(\frac{e^{-(\alpha+j\beta)t}}{-j2} + \frac{e^{-(\alpha-j\beta)t}}{+j2}\right) \quad t>0$$

$$= \frac{V_s}{L}\,\frac{1}{\beta}\,e^{-\alpha t}\underbrace{\left(\frac{e^{-j\beta t}-e^{j\beta t}}{-j2}\right)}$$

$$= \frac{1}{j2}\left(e^{j\beta t}-e^{-j\beta t}\right) = \sin\beta t$$

$\rightarrow$ For $t>0$, $\alpha = \frac{R}{2L}$, $\beta = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$

$$i_L(t) = \frac{V_L}{\beta L}\,e^{-\alpha t}\sin\beta t \cdot u(t)$$



KVL: $0 = v_C(t) + L\frac{d}{dt}\left(C\frac{dv_C}{dt}\right) + R\left(C\frac{dv_C}{dt}\right)$

$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = 0$$

divide by LC $\Rightarrow$

$$\frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = 0$$

$\downarrow \mathcal{L}$

$$s^2V_C(s) - s v_C(0) - v_C'(0) + \frac{R}{L}\left(sV_C(s) - v_C(0)\right) + \frac{1}{LC}V_C(s) = 0$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)V_C(s) = \left(s + \frac{R}{L}\right)v_C(0) + v_C'(0)$$

$$\Rightarrow V_C(s) = \frac{\left(s+\frac{R}{L}\right)v_C(0) + v_C'(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Denominator $s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\alpha s + \omega_0^2$, $\alpha = \frac{R}{2L}$, $\omega_0 = 1/\sqrt{LC}$

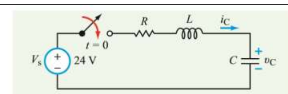
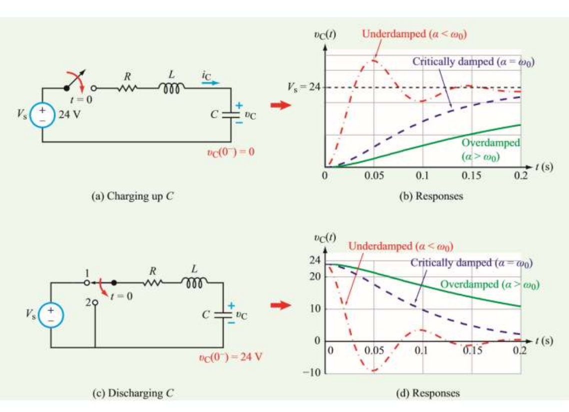


Figure 6-7: Series RLC circuit connected to a source V_s at $t = 0$. In general, the capacitor may have had an initial charge on it at $t = 0^-$, with a corresponding initial voltage $v_C(0^-)$.

6-3 Series RLC Overdamped Response ($\alpha > \omega_0$)

A key takeaway lesson from the qualitative description given in the preceding section is that after closing the switch in a series RLC circuit, the voltage across the capacitor will charge up or discharge down to equalize to the voltage across the source. In this section, we derive the differential equation for the series RLC circuit in **Fig. 6-7** and then solve it to obtain an expression for $v_C(t)$ for $t \geq 0$, with $t = 0$ designated as the time *immediately after* the switch is closed.

As noted in the preceding section, the nature of the solution for $v_C(t)$ depends on how the magnitude of the damping coefficient α compares with that of the resonant frequency ω_0. The values of the two parameters are dictated by the values of R, L, and C, per the expressions in Eq. (6.1). In the present section, we consider the case corresponding to $\alpha > \omega_0$, which is called the ***overdamped response***. The other two cases are treated in follow-up sections.



6-3.1 Differential Equation

For the circuit in **Fig. 6-7**, the KVL loop equation for $t \geq 0$ (after closing the switch) is

$$Ri_C + L\,\frac{di_C}{dt} + \underset{\frac{1}{C}\int_{-\infty}^{t} i_C(\tau)d\tau}{v_C} = V_s \qquad (\text{for } t \geq 0), \qquad (6.2)$$

where i_C and v_C are the current through and voltage across the capacitor. The capacitor may or may not have had charge on it. If it had, we denote the value of the initial voltage across it $v_C(0)$, which is the same as $v_C(0^-)$, the voltage across it before closing the switch (since the voltage across a capacitor cannot change instantaneously).

6-3.2 Solution of Differential Equation

The general solution of the second-order differential equation given by Eq. (6.5) consists of two components:

$$\upsilon_C(t) = \upsilon_{tr}(t) + \upsilon_{ss}(t), \tag{6.7}$$

where $\upsilon_{tr}(t)$ is the ***transient*** (also called ***homogeneous*** solution of Eq. (6.5) or the ***natural response*** of the RLC circuit) and $\upsilon_{ss}(t)$ is the ***steady-state*** solution (also called ***particular*** solution). The transient solution is the solution of Eq. (6.5) under source-free conditions; i.e., with $V_s = 0$, which means that $c = V_s/LC$ also is zero. Thus $\upsilon_{tr}(t)$ is the solution of

$$\upsilon''_{tr} + a\upsilon'_{tr} + b\upsilon_{tr} = 0 \qquad \textbf{(source-free)}. \tag{6.8}$$

The steady-state solution $\upsilon_{ss}(t)$ is related to the forcing function on the right-hand side of Eq. (6.5), and its functional form is similar to that of the forcing function. Since in the present case, the forcing function c is simply a constant, so is $\upsilon_{ss}(t)$. That is, $\upsilon_{ss}(t)$ is a non–time-varying constant υ_{ss} that will be determined later from initial and final conditions. Moreover, as we will see shortly, the transient component $\upsilon_{tr}(t)$ always goes to zero as $t \to \infty$ (that's why it is called *transient*). Hence, as $t \to \infty$, Eq. (6.7) reduces to

$$\upsilon_C(\infty) = \upsilon_{ss}, \tag{6.9}$$

in which case Eq. (6.7) can be rewritten as

$$\upsilon_C(t) = \upsilon_{tr}(t) + \upsilon_C(\infty). \tag{6.10}$$

Our remaining task is to determine $\upsilon_{tr}(t)$.

equations. Thus, we assume that

$$\upsilon_{tr}(t) = Ae^{st}, \tag{6.11}$$

where A and s are constants to be determined later. To ascertain that Eq. (6.11) is indeed a viable solution of Eq. (6.8), we insert the proposed expression for $\upsilon_{tr}(t)$ and its first and second derivatives in Eq. (6.8). The result is

$$s^2Ae^{st} + asAe^{st} + bAe^{st} = 0, \tag{6.12}$$

which simplifies to

$$s^2 + as + b = 0. \tag{6.13}$$

Hence, the proposed solution given by Eq. (6.11) is indeed an acceptable solution so long as Eq. (6.13) is satisfied.

The quadratic equation given by Eq. (6.13) is known as the ***characteristic equation*** of the differential equation. It has two roots:

$$s_1 = -\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 - b}\,, \tag{6.14a}$$

$$s_2 = -\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - b}\,. \tag{6.14b}$$

$$\alpha = \frac{R}{2L} = \frac{a}{2} \qquad \text{(Np/s)}, \tag{6.17a}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = b \qquad \text{(rad/s)}, \tag{6.17b}$$

the expressions given by Eq. (6.14) become

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}\,, \tag{6.18a}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}\,, \tag{6.18b}$$

The solution in the present section pertains to the overdamped case corresponding to $\alpha > \omega_0$. Under this condition, both s_1 and s_2 are real, negative numbers. Consequently, as $t \to \infty$, the first two terms in Eq. (6.16) go to zero, just as we asserted earlier.

$$\upsilon_{tr}(t) = A_1e^{s_1t} + A_2e^{s_2t} \qquad \text{for } t \ge 0, \tag{6.15}$$

where constants A_1 and A_2 are to be determined shortly.

Inserting Eq. (6.15) into Eq. (6.10) leads to

$$\upsilon_C(t) = A_1e^{s_1t} + A_2e^{s_2t} + \upsilon_C(\infty). \tag{6.16}$$

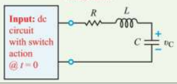
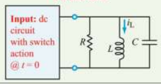
The exponential coefficients s_1 and s_2 are given by Eq. (6.14) in terms of constants a and b, both of which are defined in Eq. (6.6). By reintroducing the ***damping coefficient*** α and ***resonant frequency*** ω_0, which we defined earlier in Eq. (6.1), as

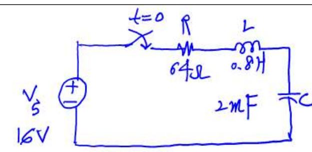
$$A_1 = \frac{\frac{1}{C}\, i_C(0) - s_2[\upsilon_C(0) - \upsilon_C(\infty)]}{s_1 - s_2}\,, \tag{6.22a}$$

$$A_2 = \frac{\frac{1}{C}\, i_C(0) - s_1[\upsilon_C(0) - \upsilon_C(\infty)]}{s_2 - s_1}\,. \tag{6.22b}$$

This concludes the general solution for the overdamped response. A summary of relevant expressions is available in **Table 6-1**.

Table 6-1: Step response of RLC circuits for $t \geq 0$.

Series RLC	Parallel RLC
	
Total Response	**Total Response**
Overdamped ($\alpha > \omega_0$) $v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + v_C(\infty)$ $A_1 = \dfrac{\frac{1}{C}\, i_C(0) - s_2[v_C(0) - v_C(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{C}\, i_C(0) - s_1[v_C(0) - v_C(\infty)]}{s_2 - s_1}\right]$	**Overdamped** ($\alpha > \omega_0$) $i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)$ $A_1 = \dfrac{\frac{1}{L}\, v_L(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{L}\, v_L(0) - s_1[i_L(0) - i_L(\infty)]}{s_2 - s_1}\right]$
Critically Damped ($\alpha = \omega_0$) $v_C(t) = (B_1 + B_2 t)e^{-\alpha t} + v_C(\infty)$ $B_1 = v_C(0) - v_C(\infty)$ $B_2 = \frac{1}{C}\, i_C(0) + \alpha[v_C(0) - v_C(\infty)]$	**Critically Damped** ($\alpha = \omega_0$) $i_L(t) = (B_1 + B_2 t)e^{-\alpha t} + i_L(\infty)$ $B_1 = i_L(0) - i_L(\infty)$ $B_2 = \frac{1}{L}\, v_L(0) + \alpha[i_L(0) - i_L(\infty)]$
Underdamped ($\alpha < \omega_0$) $v_C(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + v_C(\infty)$ $D_1 = v_C(0) - v_C(\infty)$ $D_2 = \dfrac{\frac{1}{C}\, i_C(0) + \alpha[v_C(0) - v_C(\infty)]}{\omega_d}$	**Underdamped** ($\alpha < \omega_0$) $i_L(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + i_L(\infty)$ $D_1 = i_L(0) - i_L(\infty)$ $D_2 = \dfrac{\frac{1}{L}\, v_L(0) + \alpha[i_L(0) - i_L(\infty)]}{\omega_d}$



Given that in the circuit of **Fig. 6-8(a)**, $V_s = 16$ V, $R = 64\ \Omega$, $L = 0.8$ H, and $C = 2$ mF, determine $v_C(t)$ and $i_C(t)$ for $t \geq 0$. The capacitor had no charge prior to $t = 0$.

$= v_C(0^-) = 0 = v_C(0)$

$v_C(\infty) = ?$

$v_C(\infty) = V_s = 16 [V]$

$$\alpha = \frac{R}{2L} = \frac{64}{2 \times 0.8} = 40 \text{ Np/s},$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.8 \times 2 \times 10^{-3}}} = 25 \text{ rad/s}.$$

$\alpha > \omega_0$ (overdamped case)

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -40 + \sqrt{40^2 - 25^2} = -8.8 \text{ Np/s},$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -71.2 \text{ Np/s}.$$

Prior to $t = 0$, there was no current in the circuit, and since the current through L (which is also the current through C) cannot change instantaneously, it follows that

$$i_C(0) = i_L(0) = i_L(0^-) = 0.$$

From Eq. (6.22), A_1 and A_2 are given by

$$A_1 = \frac{\frac{1}{C}\, i_C(0) - s_2[v_C(0) - v_C(\infty)]}{s_1 - s_2}$$

$$= \frac{0 + 71.2(0 - 16)}{-8.8 + 71.2} = -18.25 \text{ V},$$

$$A_2 = -\left[\frac{\frac{1}{C}\, i_C(0) - s_1[v_C(0) - v_C(\infty)]}{s_1 - s_2}\right]$$

$$= -\left[\frac{0 + 8.8(0 - 16)}{-8.8 + 71.2}\right] = 2.25 \text{ V}.$$

The total response $v_C(t)$ is then given by

$v_C(\infty)$

$$v_C(t) = [-18.25e^{-8.8t} + 2.25e^{-71.2t} + 16] \text{ V} \quad (\text{for } t \geq 0),$$