EE 101 Lecture 16, Feb 28, 2019

Quiz 8 on March 4 based on HW 8.

[1] Prob 6.1 [7] 6.22

[2] 6.3 [8] 6.25

[3] 6.7

[4] 6.12

[5] 6.16

[6] 6.18

Table 12-1: Properties of the Laplace transform ($f(t) = 0$ for $t < 0^-$).

Property	$f(t)$		$\mathbf{F}(\mathbf{s}) = \mathcal{L}[f(t)]$
1. Multiplication by constant	$K\ f(t)$	⟷	$K\ \mathbf{F}(\mathbf{s})$
2. Linearity	$K_1\ f_1(t) + K_2\ f_2(t)$	⟷	$K_1\ \mathbf{F}_1(\mathbf{s}) + K_2\ \mathbf{F}_2(\mathbf{s})$
3. Time scaling	$f(at), \quad a > 0$	⟷	$\frac{1}{a}\mathbf{F}\left(\frac{\mathbf{s}}{a}\right)$
4. Time shift	$f(t-T)\ u(t-T)$	⟷	$e^{-T\mathbf{s}}\ \mathbf{F}(\mathbf{s}), \quad T \geq 0$
5. Frequency shift	$e^{-at}\ f(t)$	⟷	$\mathbf{F}(\mathbf{s}+a)$
6. Time 1st derivative	$f' = \frac{df}{dt}$	⟷	$\mathbf{s}\ \mathbf{F}(\mathbf{s}) - f(0^-)$
7. Time 2nd derivative	$f'' = \frac{d^2 f}{dt^2}$	⟷	$\mathbf{s}^2\mathbf{F}(\mathbf{s}) - \mathbf{s}f(0^-) - f'(0^-)$
8. Time integral	$\int_{0^-}^{t} f(\tau)\ d\tau$	⟷	$\frac{1}{\mathbf{s}}\ \mathbf{F}(\mathbf{s})$
9. Frequency derivative	$t\ f(t)$	⟷	$-\frac{d}{d\mathbf{s}}\ \mathbf{F}(\mathbf{s})$
10. Frequency integral	$\frac{f(t)}{t}$	⟷	$\int_{\mathbf{s}}^{\infty} \mathbf{F}(\mathbf{s}')\ d\mathbf{s}'$

$u(t) \leftrightarrow \frac{1}{s}$

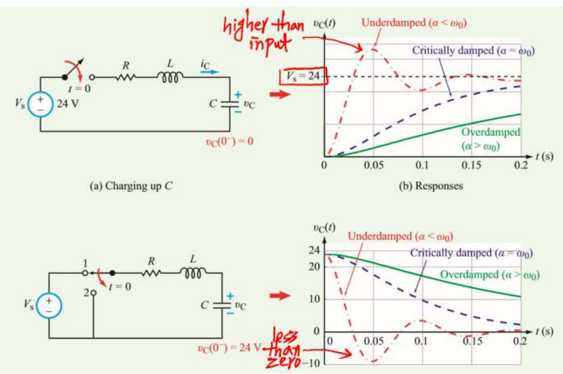
$5u(t) \leftrightarrow \frac{5}{s}$

$\sin\omega t \leftrightarrow \frac{\omega}{s^2+\omega^2}$

$\sin 2\omega t \leftrightarrow \frac{1}{2}\frac{\omega}{(\frac{s}{2})^2+\omega^2} = \frac{1}{2}\frac{\omega}{\frac{s^2}{4}+\omega^2} = \frac{2\omega}{s^2+(2\omega)^2}$

$f(t) = u(t)$, $F(s) = \frac{1}{s}$

$e^{-at}u(t) = \frac{1}{s+a}$



For series RLC circuit, its characteristic eq:

$0 = s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\frac{R}{2L}s + \omega_0^2, \quad \omega_0 = \frac{1}{\sqrt{LC}}$

$= \left(s + \frac{R}{2L}\right)^2 + \omega_0^2 - \left(\frac{R}{2L}\right)^2$

If $\omega_0 = \frac{R}{2L} (= \alpha)$, critically damped

If $\omega_0 > \frac{R}{2L} (= \alpha)$, underdamped

If $\omega_0 < \frac{R}{2L} (= \alpha)$, overdamped

where,

$\alpha = \frac{R}{2L}$ = damping coefficient, $\omega_0 = \frac{1}{\sqrt{LC}}$ = resonant freq.

$\frac{d^2 v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{V_s}{LC}$

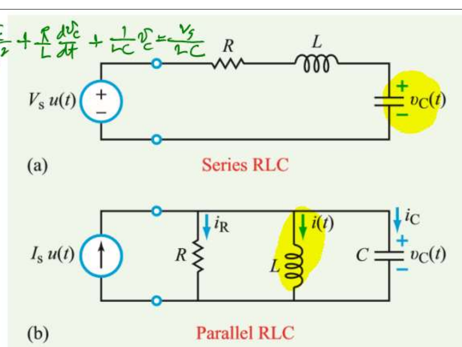


Figure 6-14: The differential equation for $v_C(t)$ of the series RLC circuit shown in (a) is identical in form to that of the current $i_L(t)$ in the parallel RLC circuit in (b).

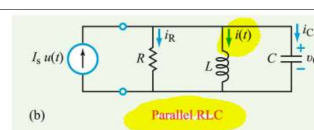
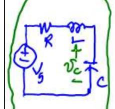


Figure 6-14: The differential equation for $v_C(t)$ of the series RLC circuit shown in (a) is identical in form to that of the current $i_L(t)$ in the parallel RLC circuit in (b).

series RLC



When expressed in terms of $v_C(t)$, the voltage common to all three passive elements, Eq. (6.51) becomes

KCL
$$\frac{v_C}{R} + i_L + C\,\frac{dv_C}{dt} = I_s. \quad (6.52)$$

$\frac{L\frac{di_L}{dt}}{R} + i_L + C\frac{d}{dt}\left(L\frac{di_L}{dt}\right) = I_s$

dividing by LC

Using $v_C = v_L = L\,di_L/dt$, and rearranging terms, leads to

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC}\,i_L = \frac{I_s}{LC},} \quad (6.53)$$

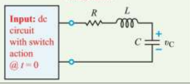
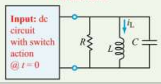
series RLC

$\frac{d^2 v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = \frac{V_s}{LC}$

which can be rewritten in the abbreviated form

$$i_L'' + a_2 i_L' + b_2 i_L = c_2, \quad (6.54)$$

Table 6-1: Step response of RLC circuits for $t \geq 0$.

Series RLC	Parallel RLC
	
Total Response	**Total Response**
Overdamped ($\alpha > \omega_0$) $v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + v_C(\infty)$ $A_1 = \dfrac{\frac{1}{C} i_C(0) - s_2[v_C(0) - v_C(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{C} i_C(0) - s_1[v_C(0) - v_C(\infty)]}{s_2 - s_1}\right]$	**Overdamped** ($\alpha > \omega_0$) $i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)$ $A_1 = \dfrac{\frac{1}{L} v_L(0) - s_2[i_L(0) - i_L(\infty)]}{s_1 - s_2}$ $A_2 = \left[\dfrac{\frac{1}{L} v_L(0) - s_1[i_L(0) - i_L(\infty)]}{s_2 - s_1}\right]$
Critically Damped ($\alpha = \omega_0$) $v_C(t) = (B_1 + B_2 t)e^{-\alpha t} + v_C(\infty)$ $B_1 = v_C(0) - v_C(\infty)$ $B_2 = \frac{1}{C} i_C(0) + \alpha[v_C(0) - v_C(\infty)]$	**Critically Damped** ($\alpha = \omega_0$) $i_L(t) = (B_1 + B_2 t)e^{-\alpha t} + i_L(\infty)$ $B_1 = i_L(0) - i_L(\infty)$ $B_2 = \frac{1}{L} v_L(0) + \alpha[i_L(0) - i_L(\infty)]$
Underdamped ($\alpha < \omega_0$) $v_C(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + v_C(\infty)$ $D_1 = v_C(0) - v_C(\infty)$ $D_2 = \dfrac{\frac{1}{C} i_C(0) + \alpha[v_C(0) - v_C(\infty)]}{\omega_d}$	**Underdamped** ($\alpha < \omega_0$) $i_L(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + i_L(\infty)$ $D_1 = i_L(0) - i_L(\infty)$ $D_2 = \dfrac{\frac{1}{L} v_L(0) + \alpha[i_L(0) - i_L(\infty)]}{\omega_d}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

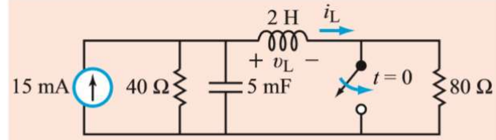
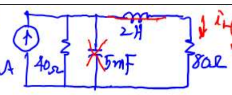


Figure E6.7

For $t<0$



$i_L(0^-) = 15\,mA \times \frac{40}{40+80} = 5\,mA$

$v_C(0^-) = v_{80\Omega} = 80 \times 5 \times 10^{-3} = 0.4V$

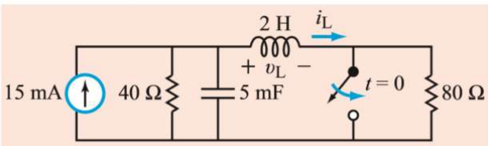
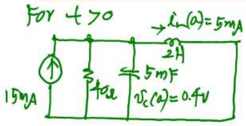


Figure E6.7

For $t>0$



$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 40 \times 5 \times 10^{-3}} = 2.5\,[Np/s]$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 5 \times 10^{-3}}} = \frac{1}{\sqrt{0.01}} = 10\,[rad/s]$

$\alpha < \omega_0$ (underdamped)

$i_L(\infty) = 15\,mA$, $v_L(0) = v_C(0) = 0.4\,[V]$

$i_L(t) = e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + i_L(\infty)$

$D_1 = i_L(0) - i_L(\infty) = (5 - 15)[mA] = -10\,mA$

$D_2 = \dfrac{\frac{1}{L} v_L(0) + \alpha[i_L(0) - i_L(\infty)]}{\omega_d} = \dfrac{\frac{1}{2}(0.4) + 2.5(-10mA)}{9.68}$
$= (0.2 - 2.5 \times 10^{-3})/9.68$
$= 18.08$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^2 - 0.25^2} = 9.68\,[rad/s]$

$i_L(t) = 15 - (10 \cos 9.68t + 18.08 \sin 9.68t)\, e^{-2.5t}$
$\times u(t)\ [mA]$

At $t = \infty$, the energy stored in the 2H inductor is

$E_L(\infty) = \frac{1}{2} L i_L^2(\infty) = \frac{1}{2}(2)[15 \times 10^{-3}]^2 = 225 \times 10^{-6}$ Joule

$v_C(\infty) = v_L(\infty) = 0\,[V]$

Exercise 6-8: In the parallel RLC circuit shown in Fig. 6-14(b), how much energy will be stored in L and C at $t = \infty$?

Answer: $w_L = \frac{1}{2} L I_s^2$, $w_C = 0$. (See CAD)

6-8 General Solution for Any Second-Order Circuit with dc Sources

According to the material covered in the preceding sections, series and parallel RLC circuit share a common set of characteristics. An RLC circuit is characterized by a resonant frequency ω_0 and a damping coefficient α, and when driven by a sudden dc excitation, the circuit exhibits a response that decays exponentially as $e^{-\alpha t}$, and it may or may not contain an oscillatory variation, depending on whether ω_0 is or is not larger than α in magnitude, respectively. These characteristics arise from the interplay between energy storage and energy dissipation. During the operation of the RLC circuit, energy is exchanged between the two storage elements—the capacitor and the inductor—through the resistor. Dissipation is governed by $e^{-\alpha t}$, which we can redefine as $e^{-t/\tau}$, with

$$\tau = \frac{1}{\alpha} \quad \text{(s)}. \tag{6.58}$$

In this alternative form, the decay rate is specified by the time constant τ. If τ is short (rapid decay) in comparison with the duration of a single oscillation period T, where $T = 2\pi/\omega_d$, it means that energy burns away too quickly to generate an oscillation. This is the overdamped case. On the other hand, if τ is sufficiently long (slow decay) in comparison with T,

Step 1: Develop a second-order differential equation for $x(t)$, for $t \geq 0$. Express the equation in the general form

$$x'' + ax' + bx = c, \tag{6.59}$$

where a, b, and c are constants.

Step 2: Determine the values of α and ω_0:

$$\alpha = \frac{a}{2}, \qquad \omega_0 = \sqrt{b}. \tag{6.60}$$

Step 3: Determine whether the response $x(t)$ is overdamped, critically damped, or underdamped, and write down the expression corresponding to that case from the following general solution:

General Solution

Overdamped ($\alpha > \omega_0$)

$$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)], \quad (\text{for } t \geq 0) \tag{6.61a}$$

Critically Damped ($\alpha = \omega_0$)

$$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)], \quad (\text{for } t \geq 0) \tag{6.61b}$$

Underdamped ($\alpha < \omega_0$)

$$x(t) = [e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + x(\infty)], \quad (\text{for } t \geq 0) \tag{6.61c}$$

Table 6-2: General solution for second-order circuits for $t \geq 0$.

$x(t)$ = unknown variable (voltage or current)	
Differential equation:	$x'' + ax' + bx = c$
Initial conditions:	$x(0)$ and $x'(0)$
Final condition:	$x(\infty) = \frac{c}{b}$
$\alpha = \frac{a}{2}$	$\omega_0 = \sqrt{b}$
Overdamped Response $\alpha > \omega_0$	
$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]\, u(t)$	
$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$	$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$A_1 = \dfrac{x'(0) - s_2[x(0) - x(\infty)]}{s_1 - s_2}$	$A_2 = -\left[\dfrac{x'(0) - s_1[x(0) - x(\infty)]}{s_1 - s_2}\right]$
Critically Damped $\alpha = \omega_0$	
$x(t) = [(B_1 + B_2 t)e^{-\alpha t} + x(\infty)]\, u(t)$	
$B_1 = x(0) - x(\infty)$	$B_2 = x'(0) + \alpha[x(0) - x(\infty)]$
Underdamped $\alpha < \omega_0$	
$x(t) = [e^{-\alpha t}(D_1 \cos\omega_d t + D_2 \sin\omega_d t) + x(\infty)]\, u(t)$	
$D_1 = x(0) - x(\infty)$	$D_2 = \dfrac{x'(0) + \alpha[x(0) - x(\infty)]}{\omega_d}$
$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	

Exercise 6-9: Develop an expression for $i_C(t)$ in the circuit of **Fig. E6.9** for $t \geq 0$.

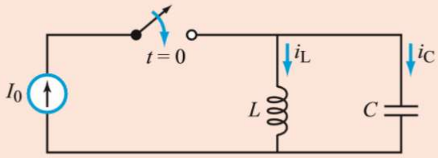


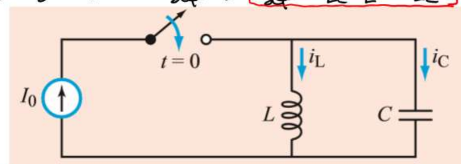
Figure E6.9

Answer: $i_C(t) = I_0 \cos \omega_0 t$, with $\omega_0 = 1/\sqrt{LC}$. This is an LC *oscillator* circuit in which dc energy provided by the current source is converted into ac energy in the LC circuit. (See CAD)

$i_L(0^-) = 0 \quad v_C(0^-) = 0 \qquad i_L(\infty) = I_0, \; v_C(\infty) = 0$

KCL: $I_0 = i_L + i_C$, $\quad i_C = C\frac{dv_C}{dt} = C\frac{dv_L}{dt} = C\left(L\frac{di_L}{dt}\right) = LC\frac{d^2 i_L}{dt^2}$

$\Rightarrow I_0 = i_L + LC\frac{d^2 i_L}{dt^2} \Rightarrow \boxed{\frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = \frac{I_0}{LC}}$



$\alpha = \frac{1}{2RC} = 0 \quad \omega_0 = \frac{1}{\sqrt{LC}},$

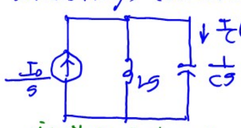
$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0$

$i_L(t) = D_1 \cos \omega_0 t + D_2 \sin \omega_0 t + I_0$

$D_1 = i_L(0) - i_L(\infty) = 0 - I_0 = -I_0, \; D_2 = \frac{1}{L} v_L(0) = 0$

$i_L(t) = I_0(1 - \cos \omega_0 t)$, $i_C(t) = I_0 - I_L(t) = I_0 \cos \omega_0 t$

Alternatively, (direct solution method)



circuit in s-domain

$i_L(0) = 0, \; v_C(0) = 0$

$I_C(s) = \frac{I_0}{s}\left(\frac{Ls}{Ls + \frac{1}{Cs}}\right)$

current division law

$= \frac{I_0}{s} \, \frac{LCs^2}{LCs^2 + 1}$

$= \frac{I_0}{s} \, \frac{s^2}{(s^2 + \frac{1}{LC})} = \frac{I_0 s}{s^2 + \frac{1}{LC}}$

$= I_0 \frac{s}{s^2 + \omega_0^2}, \; \omega_0 = \frac{1}{\sqrt{LC}}$

$\mathcal{L}^{-1}$

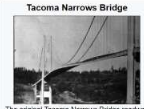
$\boxed{i_C(t) = I_0 \cos \omega_0 t}$

The **1940 Tacoma Narrows Bridge**, the first Tacoma Narrows Bridge, was a suspension bridge in the U.S. state of Washington that spanned the Tacoma Narrows strait of Puget Sound between Tacoma and the Kitsap Peninsula. It opened to traffic on July 1, 1940, and dramatically collapsed into Puget Sound on November 7 of the same year. At the time of its construction (and its destruction), the bridge was the third longest suspension bridge in the world in terms of main span length, behind the Golden Gate Bridge and the George Washington Bridge.

Construction on the bridge began in September 1938. From the time the deck was built, it began to move vertically in windy conditions, which led to construction workers giving the bridge the nickname **Galloping Gertie**. The motion was observed even when the bridge opened to the public. Several measures aimed at stopping the motion were ineffective and the bridge's main span finally collapsed under 40-mile-per-hour (64 km/h) wind conditions the morning of November 7, 1940.

Following the collapse, the United States' involvement in World War II delayed plans to replace the bridge. The

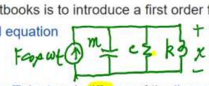
Tacoma Narrows Bridge



The original Tacoma Narrows Bridge roadway twisted and vibrated violently under 40-mile-per-hour (64 km/h) winds on the day of the collapse

Usually, the approach taken by those physics textbooks is to introduce a first order forced oscillator, defined by the second-order differential equation

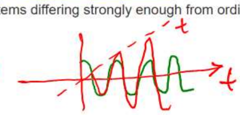
$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F\cos(\omega t) \qquad \textit{(eq. 1)}$$



where m, c and k stand for the mass, damping coefficient and stiffness of the linear system and F and ω represent the amplitude and the angular frequency of the exciting force. The solution of such ordinary differential equation as a function of time *t* represents the displacement response of the system (given appropriate initial conditions). In the above system resonance happens when ω is approximately $\omega_r = \sqrt{k/m}$, i.e. ω_r is the natural (resonant) frequency of the system. The actual vibration analysis of a more complicated mechanical system—such as an airplane, a building or a bridge—is based on the linearization of the equation of motion for the system, which is a multidimensional version of equation (**eq. 1**). The analysis requires eigenvalue analysis and thereafter the natural frequencies of the structure are found, together with the so-called *fundamental modes* of the system, which are a set of independent displacements and/or rotations that specify completely the displaced or deformed position and orientation of the body or system, i.e., the bridge moves as a (linear) combination of those basic deformed positions.

However, to some degree the debate is due to the lack of a commonly accepted precise definition of resonance. Billah and Scanlan[1] provide the following definition of resonance "In general, whenever a system capable of oscillation is acted on by a periodic series of impulses having a frequency equal to or nearly equal to one of the natural frequencies of the oscillation of the system, the system is set into oscillation with a relatively large amplitude." They then state later in their paper "Could this be called a resonant phenomenon? It would appear not to contradict the qualitative definition of resonance quoted earlier, if we now identify the source of the periodic impulses as *self-induced*, the wind supplying the power, and the motion supplying the power-tapping mechanism. If one wishes to argue, however, that it was a case of *externally forced linear resonance*, the mathematical distinction ... is quite clear, self-exciting systems differing strongly enough from ordinary linear resonant ones."

$t\cos\omega t$



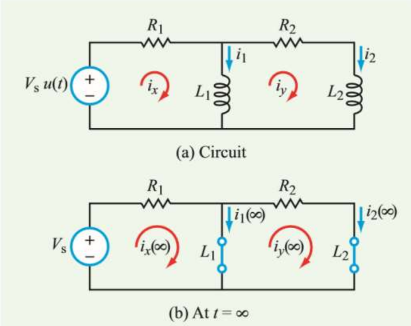


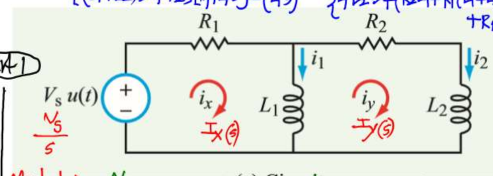
Figure 6-17: Circuit for Example 6-11.

Example 6-11: Two-Inductor Circuit

From (3) & (4) $\left[(L_1+L_2)s+R_2\right]I_y(s) = L_1 s\,\dfrac{\left[\frac{V_s}{s}+L_1 s\,I_y(s)\right]}{R_1+L_1 s}$

Part 2 $\Rightarrow \left[(L_1+L_2)s+R_2\right]\left[R_1+L_1 s\right]I_y(s) = L_1 V_s + (L_1 s)^2 I_y(s)$

$$I_y(s) = \frac{L_1 V_s}{\left[(L_1+L_2)s+R_2\right]\left[R_1+L_1 s\right]-(L_1 s)^2} = \frac{L_1 V_s}{\{L_1L_2 s^2+(R_2L_1+R_1(L_1+L_2))s+R_1R_2\}} \quad (5)$$



Mesh 1 : $-\frac{V_s}{s}+R_1 I_x(s) + L_1 s\,(I_x(s)-I_y(s)) = 0 \quad (1)$

$L_1 s\,(I_y(s)-I_x(s)) + R_2 I_y(s) + L_2 s\,I_y(s) = 0 \quad (2)$

From (1) $(R_1+L_1 s)I_x(s) = \frac{V_s}{s} + L_1 s\,I_y(s)$

$I_x(s) = \left(\frac{V_s}{s}+L_1 s\,I_y(s)\right)/(R_1+L_1 s) \quad (3)$

(3)→(2) $(L_1 s+R_2+L_2 s)I_y(s) = L_1 s\,I_x(s) \quad (4)$

$$I_y(s) = \frac{L_1 V_s}{L_1L_2 s^2+(R_2L_1+R_1(L_1+L_2))s+R_1R_2}$$

$$= \frac{V_s/L_2}{s^2+\left(\frac{R_2}{L_2}+R_1\left(\frac{L_1+L_2}{L_1L_2}\right)\right)s+\frac{R_1R_2}{L_1L_2}} \quad (6)$$

(6) → (3) will find $I_x(s)$.

Determine $i_1(t)$ and $i_2(t)$ in the circuit of **Fig. 6-17** for $t \geq 0$. The component values are $V_s = 1.4$ V, $R_1 = 0.4\ \Omega$, $R_2 = 0.3\ \Omega$, $L_1 = 0.1$ H, and $L_2 = 0.2$ H.

$$I_y(s) = \frac{1.4/0.2\ (=7)}{s^2+(1.5+0.4\times15)s+6} = \frac{7}{s^2+7.5s+6}$$

$$= \frac{7}{(s+0.91)(s+6.59)} = \frac{1.23}{s+0.91} + \frac{-1.23}{s+6.59}$$

$\downarrow \mathcal{L}^{-1}$

$i_y(t) = 1.23\left(e^{-0.91t} - e^{-6.59t}\right)$ same as (6.99) on p367

Exercise 6-10: For the circuit in **Fig. E6.10**, determine $i_C(t)$ for $t \geq 0$.

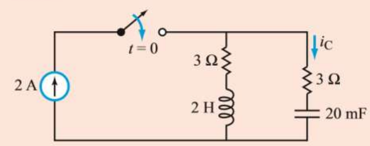


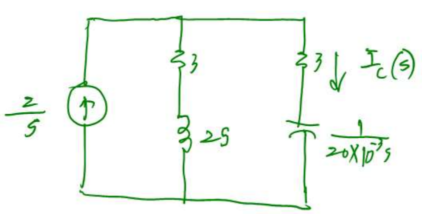
Figure E6.10

$i_L(0)=0$
$v_C(0)=0$

Answer: $i_C(t) = 2e^{-1.5t}\cos 4.77t$ A. (See CAD)

Example 6-12: Second-Order Op-Amp Circuit

Determine $i_L(t)$ in the op-amp circuit of **Fig. 6-18(a)** for $t \geq 0$. Assume $V_s = 1$ mV, $R_1 = 10$ kΩ, $R_2 = 1$ MΩ, $R_3 = 100\ \Omega$, $L = 5$ H and $C = 1\ \mu$F.



$$I_C(s) = \frac{2}{s}\left[\frac{3+2s}{3+2s+3+\frac{1}{20\times10^{-3}s}}\right]$$

$$= \frac{2}{s}\,\frac{(3+2s)\,20\times10^{-3}s}{(6+2s)(20\times10^{-3}s)+1} = 2\,\frac{20\times10^{-3}(3+2s)}{40\times10^{-3}s^2+120\times10^{-3}s+1}$$

$$= 2\,\frac{20\times10^{-3}(3+2s)}{(40\times10^{-3})s^2+120\times10^{-3}s+1}$$

$\div(40\times10^{-3})$

$$\frac{1\,(3+2s)}{s^2+3s+\frac{1}{40\times10^{-3}}(=25)} = I_C(s)$$

$\omega_0^2 \quad \omega_0=5$

$\alpha = \frac{3}{2}=1.5, \quad \alpha<\omega_0$ underdamped $\Downarrow$ sinusoidal

$$I_C(s) = \frac{2s+3}{s^2+3s+25} = \frac{2s+3}{(s+1.5)^2+(25-1.5^2)} \qquad (25-1.5^2)=\omega_d^2$$

$$= \frac{2s+3}{(s+1.5+j\omega_d)(s+1.5-j\omega_d)}$$

$$= \frac{A_1=1}{s+1.5+j\omega_d} + \frac{A_2=A_1^*=1}{s+1.5-j\omega_d}$$

$\omega_d^2 = \omega_0^2-\alpha^2$

$\omega_d = \sqrt{\omega_0^2-\alpha^2}$

$$\mathcal{L}^{-1}\left(\frac{1}{s+1.5+j\omega_d}+\frac{1}{s+1.5-j\omega_d}\right)$$

$$= e^{-1.5t}e^{-j\omega_d t} + e^{-1.5t}e^{+j\omega_d t}$$

$$= e^{-1.5t}\left(e^{+j\omega_d t} + e^{-j\omega_d t}\right)$$

$$= e^{-1.5t}\, 2\, \underbrace{\left(\frac{e^{+j\omega_d t}+e^{-j\omega_d t}}{2}\right)}_{=\cos\omega_d t}$$

$$= 2e^{-1.5t}\underbrace{\cos\omega_d t}_{\cos 4.77t}$$

$$\omega_d = \sqrt{\underset{5^2}{\omega_0^2} - \underset{1.5^2}{\alpha^2}} = 22.75$$

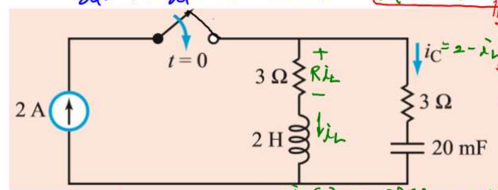
$$= 4.77$$

$$3i_L + 2\frac{di_L}{dt} = 3(2-i_L) + \frac{1}{20\times10^{-3}}\int_0^t (2-i_L(\tau))\,d\tau$$

$$\frac{d}{dt} \rightarrow 3\frac{di_L}{dt} + 2\frac{d^2 i_L}{dt^2} = -3\frac{di_L}{dt} + 50\,(2-i_L)$$

$$2\frac{d^2 i_L}{dt^2} + 6\frac{di_L}{dt} + 50\,i_L = 100 \Rightarrow \boxed{\frac{d^2 i_L}{dt^2} + 3\frac{di_L}{dt} + 25 i_L = 50}$$

$$= \frac{i_L}{0.04} = \frac{1}{LC}i_L = \omega_0^2 i_L$$



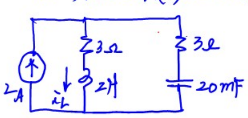
$i_L(0)=0$ $v_L(0)=0,\ i_L(\infty)=2$

$\alpha = \frac{3}{2}$ $\omega_0 = 0.2$, $\alpha > \omega_0$ (overdamped)

$$i_L(t) = A e^{s_1 t} + A_2 e^{s_2 t} + i_L(\infty)$$

$$A_1 = \frac{\frac{1}{2}v_L(0) - s_2(0-2)}{s_1 - s_2} \quad A_2 = \frac{\frac{1}{2}v_L(0) - s_1(0-2)}{s_2 - s_1}$$

To find $v_L(0)$, consider



$$3i_L(0) + v_L(0) = 3(2 - i_L(0)) + v_c(0)$$

$$v_c(0) = v_c(0-) = 0,\ i_L(0) = i_L(0-) = 0$$

$$\Rightarrow \boxed{v_L(0) = 6}$$

From $\frac{d^2 i_L}{dt^2} + 3\frac{di_L}{dt} + 25 i_L = 0$

$$s^2 + 3s + 25 = 0 \Rightarrow s_1, s_2 = \frac{-3 \pm \sqrt{3^2 - 4\times25}}{2}$$

$$= \frac{-3 \pm j\sqrt{91}}{2}$$

$$s_1 - s_2 = j\sqrt{91},\ s_2 - s_1 = -j\sqrt{91}$$

$$A_1 = \frac{\frac{1}{2}v_L(0) - s_2(0-2)}{s_1 - s_2} = \frac{3 - \frac{-3-j\sqrt{91}}{2}(0-2)}{j\sqrt{91}} = -1$$

$$A_2 = \frac{\frac{1}{2}v_L(0) - s_1(0-2)}{s_2 - s_1} = \frac{3 - \frac{-3+j\sqrt{91}}{2}(0-2)}{-j\sqrt{91}} = -1$$

thus,

$$i_L(t) = A_1 e^{+s_1 t} + A_2 e^{+s_2 t} + \underset{2}{i_L(\infty)}$$

$$= 2 - e^{+\left(\frac{-3+j\sqrt{91}}{2}\right)t} - e^{+\left(\frac{-3-j\sqrt{91}}{2}\right)t}$$

$$= 2 - e^{-1.5t}\underbrace{\left(e^{+j\frac{\sqrt{91}}{2}} + e^{-j\frac{\sqrt{91}}{2}}\right)}_{2\cos\frac{\sqrt{91}}{2}t}$$

$$= 2\left(1 - e^{-1.5t}\cos\frac{\sqrt{91}}{2}t\right),\ \text{for } t>0$$

$$= 2\left(1 - e^{-1.5t}\cos\left(\frac{\sqrt{91}}{2}\right)t\right)u(t) \qquad \frac{\sqrt{91}}{2} = 4.77$$

$$i_C(t) = 2 - i_L(t) = 2e^{-1.5t}\cos\left(\frac{\sqrt{91}}{2}\right)t\ \ u(t)$$