EE 101 Lecture 11-12, Feb 14, 2019
Quiz 6 on Feb 19 based on HW 6.

Topics covered so far:
- Ohm’s Law, KCL & KVL
- Equivalent circuits, Norton’s & Thevenin’s
- Node analysis, Mesh analysis
- BJT circuits, Op amp circuits
- MOS circuits
- RC & RL circuits

HW #6 (for Quiz 6)

[1] Prob 5.1
[2] 5.17
[3] 5.3
[4] 5.18
[5] 5.4
[6] 5.19
[7] 5.15
[8] 5.16

Express $v_o$ in terms of $v_i$ and $v'$.  

![Diagram of circuit](image)

Since $i_x = 0$, we have:
\[ i_x = i_0 - i_2 = \frac{v_0 - v_2}{R_2} - \frac{v_2}{R_2} = \frac{v_0 - 2v_2}{R_2} \]

and:
\[ v_{i2} = v_0 - v_2 = \left(\frac{R_1 + R_2}{R_2}\right)v_0 - v_2 \]  \hspace{1cm} (4.59)

Op amp 1 is a standard inverting amplifier, so we can use Table 4.4(a) to obtain:
\[ v'_{i2} = \left(\frac{R_1}{R_2}\right)v_{i2} = -v_{i2} \]

Op amp 2 is an inverting summing amplifier (Table 4.4(a) with output:
\[ v_0 = R_2 i_0 + v'_{i2} \]
\[ = R_2 i_0 + R_2 (-v_{i2}) \]
\[ = R_2 (i_0 - i_{i2}) = R_2 \left(\frac{v_0 - v_2}{R_2}\right) \]  \hspace{1cm} (4.51)
Transistor Invention (1948) to Integrated Circuit (1958)

First Successful Operation of MOS Transistor

Dawon Kahng (May 4, 1931 - May 13, 1992)

- SNU (BS), Ohio State Univ. (Ph.D. 1959)
- Dr. Kahng, with M. Atalla, fabricated a MOSFET using a gate insulator formed from high quality SiO2 grown by a new high-pressure steam oxidation process at Bell Labs (1960)
- First successful demonstration of MOSFET was a major milestone in semiconductor technology
- Invented in 1997 a field effect memory, the first nonvolatile silicon memory (floating gate memory)
- Became Founding President of NEC, Princeton, NJ in 1988

4-11 The MOSFET as a Voltage-Controlled Current Source

In earlier sections, we demonstrated how op amps can be used to build buffers and amplifiers. We now examine how to realize the same outcome using MOSFETs. The simplest model of a MOSFET, which stands for metal-oxide semiconductor field-effect transistor, is shown in Fig. 4-27(a). The vast majority of commercial computer processors are built with MOSFETs; as mentioned in Technology Brief 1 on nanotechnology, a 2010 Intel Core processor contains over 1 billion independent MOSFETs. A MOSFET has three terminals: the gate (G), the source (S), and the drain (D). Actually, it has a fourth terminal, namely its body (B), but we will ignore it for now because for many applications it is simply connected to the ground terminal. The circuit symbol for the MOSFET may look somewhat unusual, but it is actually a stylized depiction of the physical cross section of a real MOSFET. In a real MOSFET, the gate...
Section 4.11: MOSFET

4.59 In Example 4.9, we analyzed a common-source amplifier without a load resistance. Consider the amplifier in Fig. P4.59; it is identical to the circuit in Fig. 4.31, except that we have added a load resistor $R_L$. Obtain an expression for $v_{out}$ as a function of $v_D$.

4.60 Determine $v_{out}(t)$ as a function of $v_D(t)$ for the circuit in Fig. P4.60. Assume $V_{DD} = 2.5$ V.

(b) With MOSFET Buffer

For the circuit in Fig. 4.31(c), in which the MOSFET has been replaced with its equivalent circuit, KVL gives

$$-i_s + v_{DS} + v_{out} = 0.$$ 

Also,

$$v_{out} = i_D R_L = g_D R_L v_s.$$ 

Simultaneous solution of the two equations gives

$$v_{out} = \frac{g_D R_L}{1 + g_D R_L} v_s.$$ 

With $g = 10 \text{ A/V}$ and in order for $v_{out}$ to be no less than $0.99v_s$, it is necessary that $R_L \geq 9.9 \Omega$.

which is three orders of magnitude smaller than the requirement for the unbuffered circuit.
RC and RL First-Order Circuits

**Objectives**

1. Use mathematical functions to describe several linear and non-linear functions.
2. Define the electrical properties of a capacitor, including its a capacitance and energy stored.
3. Write expressions for voltage across a capacitor.
4. Define the electrical properties of an inductor, including its inductance and energy stored.
5. Create multiple equations when combined to solve problems.

**Contents**

1. Introduction to RC Circuits
2. First-Order RC Circuits
3. Second-Order RC Circuits
4. capacitor 
5. inductor 
6. Energy Storage in Capacitors
7. Energy Storage in Inductors
8. RC and RL Circuits

**Example Calculation**

\[ V(t) = \frac{1}{C} \int i(t) dt \]

**Graphical Representation**

- Capacitor as a fundamental device
- Circuit diagrams for RC and RL networks
- Energy storage curves for capacitors and inductors
- Calculations for capacitance and inductance

**Formulas**

- Capacitance: \[ C = \frac{Q}{V} \]
- Energy Stored: \[ U = \frac{1}{2} CV^2 \]
- Inductance: \[ L = \frac{V}{I} \]
- Energy Stored: \[ U = \frac{1}{2} LI^2 \]

**Exercises**

- Solve for the voltage across a capacitor using the current values.
- Calculate the energy stored in an inductor given its current and voltage.
- Determine the time constant for an RC circuit.

**Questions for Review**

1. Explain the difference between a capacitor and an inductor in terms of energy storage.
2. How does the time constant affect the response of an RC circuit?
3. What is the significance of the parallel combination of a capacitor and an inductor in electrical circuits?
The $C$-relationship expressed by Eq. (5.25) conveys a very important condition, namely:

- The voltage across a capacitor cannot change instantaneously, but the current can.

This assertion is supported by the observation that if it were to change its voltage too quickly, the electric field in an instant, it would disrupt the charge and voltage distribution within the capacitor, thus causing a change in its instantaneous properties. Another example of Eq. (5.25) relates to the behavior of a capacitor under a constant voltage across it, where $V = V_0$ is constant, but $Q$ changes. Such behavior is characteristic of an open circuit, through which no current flows, and if a voltage suddenly applies across it, then:

- Under these conditions, a capacitor behaves like an open circuit.

**Figure 5.19:** Voltage-division rules for (a) in-series resistors and (b) in-series capacitors.

Derive the equivalent voltage-division equation for the series capacitors $C_1$ and $C_2$ in Fig. 5.19(b). Assume that the capacitors had no charge on them before they were connected to $V_0$.

**Solution:** From the standpoint of the source $V_0$, it "sees" an equivalent, single capacitor $C$ given by the series combination of $C_1$ and $C_2$, namely

$$ C = \frac{C_1 C_2}{C_1 + C_2} \quad \text{(5.44)} $$

The voltage across $C$ is $V$. The law of conservation of energy requires that the energy that would be stored in the equivalent capacitor $C$ be equal to the sum of the energies stored in $C_1$ and $C_2$. Hence, application of Eq. (5.29) gives

$$ \frac{1}{2} C V^2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2. \quad \text{(5.45)} $$

Upon replacing $C$ with the expression given by Eq. (5.44) and replacing the source voltage with $V_0 = V_1 + V_2$, we have

$$ \frac{1}{2} \left( \frac{C_1 C_2}{C_1 + C_2} \right) (V_1 + V_2)^2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2. \quad \text{(5.46)} $$

The same analysis applies to the equivalent conductance $G$ of a series combination of conductances $G_1$ and $G_2$.
Exercise 5.18: Determine \( C_{eq} \) and \( v_{0}(0) \) at terminals (a, b) for the circuit in Fig. 5.18 given that \( C_1 = 6 \) \( \mu \)F, \( C_2 = 4 \) \( \mu \)F, \( C_3 = 8 \) \( \mu \)F, and the initial voltages on the three capacitors are \( v_1(0) = 5 \) V and \( v_2(0) = v_3(0) = 10 \) V, respectively.

![Figure 5.18](image1)

Answer: \( C_{eq} = 4 \) \( \mu \)F, \( v_{0}(0) = 15 \) V. (See \( \Box \))

5.18 Reduce the circuit in Fig. 5.18 into a single equivalent capacitor at terminals (a, b). Assume that all initial voltages are zero at \( t = 0 \).

![Figure 5.18](image2)

5.18 Reduce the circuit in Fig. 5.18 into a single equivalent capacitor at terminals (a, b). Assume that all initial voltages are zero at \( t = 0 \).

![Figure 5.18](image3)

*5.19 For the circuit in Fig. 5.19, find \( C_{eq} \) at terminals (a, b). Assume all initial voltages to be zero.

![Figure 5.19](image4)

![Figure 5.19](image5)

*5.19 For the circuit in Fig. 5.19, find \( C_{eq} \) at terminals (a, b). Assume all initial voltages to be zero.

![Figure 5.19](image6)
5.3 Inductors

Any current-carrying conductor, whether straight or coiled, forms an inductor. A current produces a magnetic field, which stores energy that can be released later in the form of another current. Also, since every wire acts like an inductor, we have small amounts of stray inductance in every circuit. Fortunately, this can be ignored except at extremely high frequencies (microwave band).

Inductors exhibit a number of useful properties, including magnetic coupling and electromagnetic induction. They are employed in microphones and loudspeakers, magnetic relays and sensors, theft detection devices, and motors and generators, and they provide wireless power transmission and data communication (albeit over relatively short distances).

Capacitors and inductors constitute a canonical pair of devices. Whereas a capacitor can store energy through the electric field induced by the voltage imposed across its terminals, an inductor can store magnetic energy through the magnetic field induced by the current flowing through its wires.

\[ V = \frac{\partial A}{\partial t} \]

5.3.1 Electrical Properties

According to Faraday's law, if the magnetic-flux linkage in an inductor (or circuit) changes with time, it induces a voltage \( V \) across the inductor’s terminals given by

\[ V = \frac{d\phi}{dt} \]

In view of Eq. (5.51),

\[ V = \frac{d}{dt} \left( \frac{\partial L}{\partial t} \right) = L \frac{d}{dt} \left( \frac{d i}{dt} \right) \]

This \( i-t \) relationship adheres to the passive sign convention introduced earlier for resistors and capacitors. If the direction of \( i \) is into the (+) voltage terminal of the inductor (Fig. 5-22), then the inductor is receiving power. Also, the same logic that led us earlier to the conclusion that the voltage across a capacitor cannot change instantaneously leads us now to the conclusion:

The current through an inductor cannot change instantaneously, but the voltage can.

\[ i(t) = i(0) + \frac{1}{L} \int_0^t v \, dt' \]  

where \( i(0) \) is an initial reference point in time.

The power delivered to the inductor is given by

\[ p(t) = vi = i \frac{d}{dt} \left( \frac{d}{dt} \right) \]

and as with the resistor and the capacitor, the sign of \( p \) determines whether the inductor is delivering power \( (p > 0) \) or absorbing power \( (p < 0) \). The accumulation of power over time constitutes the storage of energy. The magnetic energy stored in an inductor is

\[ w(t) = \frac{1}{2} L \int_0^t i^2 \, dt' \]  

which yields

\[ w(t) = \frac{1}{2} L \int_0^t v \, dt' = 0 \]  

Figure 5-20: The inductance of a solenoid of length \( l \) and cross-sectional area \( A \) is \( L = \mu_0 N^2 A / l \), where \( N \) is the number of turns and \( \mu_0 \) is the magnetic permeability of the core material.
5.31 The values of all inductors in the circuit of Fig. P5.31 are in millihenrys. Determine $I_{eq}$.

$\text{Figure P5.31: Circuit for Problem 5.31.}$

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5.15 Determine voltages $v_1$ to $v_3$ in the circuit of Fig. P5.15 under dc conditions.

$\text{Figure P5.15: Circuit for Problem 5.15.}$

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$\text{Figure P5.15: Circuit for Problem 5.15.}$

5.28 For the circuit in Fig. P5.28, determine the voltage across $C$ and the currents through $L_1$ and $L_2$ under dc conditions.

$\text{Figure P5.28: Circuit for Problem 5.28.}$
5.29 For the circuit in Fig. P5.29, determine the voltages across $C_1$ and $C_2$ and the currents through $I_1$ and $I_2$ under dc conditions.

![Figure P5.29: Circuit for Problem 5.29.](image)

5.16 Determine the voltages across the two capacitors in the circuit of Fig. P5.16 under dc conditions.

![Figure P5.16: Circuit for Problem 5.16.](image)