EE 101 Lecture 11-12, Feb 14, 2019

Quiz 6 on Feb 19 based on HW 6.

Topics covered so far

- Ohm's Law, KCL & KVL
- equivalent circuits, Norton's & Thevenin's
- Nodal analysis, Mesh analysis
- BJT circuits, Op Amp circuits
- MOS circuits
- $\boxed{\text{RC \& RL circuits}}$

HW # 6 (for Quiz 6)

[1] Prob 5.1

[2] 5.3

[3] 5.4

[4] 5.15

[5] 5.16

[6] 5.17

[7] 5.18

[8] 5.19

[Prob] Express v_o in terms of v_1 & v_2

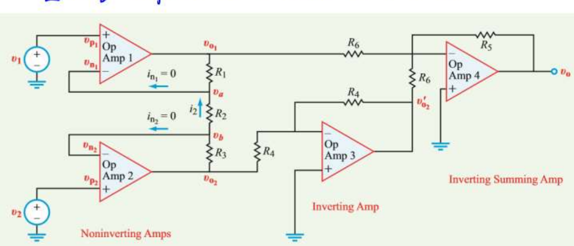
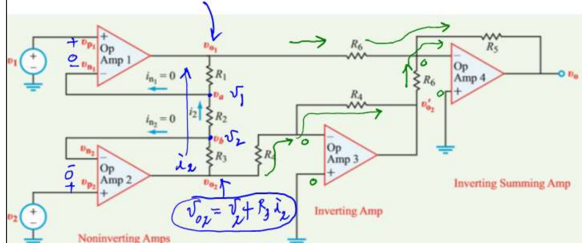


Figure 4-20: Example 4-6.

$v_{o_1} = v_1 - R_1 i_2$



$v_{o_2} = v_2 + R_3 i_2$

$$i_2 = \frac{v_2 - v_1}{R_2}$$

Since $i_{n_1} = i_{n_2} = 0$ (op-amp current constraint),

$$i_2 = \frac{v_b - v_a}{R_2} = \frac{v_2 - v_1}{R_2},$$

and

$$v_{o_2} - v_{o_1} = i_2(R_1 + R_2 + R_3)$$
$$= \left(\frac{R_1 + R_2 + R_3}{R_2}\right)(v_2 - v_1). \quad (4.50)$$

Op amp 3 is a standard inverting amplifier, so we can use Table 4-3(c) to obtain

$$v'_{o_2} = -\left(\frac{R_4}{R_4}\right) v_{o_2} = -v_{o_2}.$$

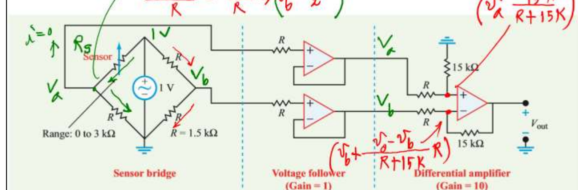
Op amp 4 is an inverting summing amplifier (Table 4-3(c)) with output

$$v_o = -\frac{R_5}{R_6}(v_{o_1} + v'_{o_2})$$
$$= -\frac{R_5}{R_6}(v_{o_1} - v_{o_2})$$
$$= \frac{R_5}{R_6}(v_{o_2} - v_{o_1}) = R_5\left(\frac{R_1 + R_2 + R_3}{R_6 R_2}\right)(v_2 - v_1). \quad (4.51)$$

$$\frac{1 - v_a}{R_s} = \frac{v_a}{R} \Rightarrow R(1 - v_a) = R_s v_a$$
$$R = (R + R_s) v_a, \quad v_a = \frac{R}{R + R_s}$$

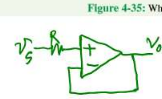
$$\frac{1 - v_b}{R} = \frac{v_b}{R} \Rightarrow \left(v_b = \frac{1}{2}\right)$$

$$\left(v_a \frac{15K}{R + 15K}\right)$$



$$\left(v_b + \frac{v_o - v_b}{R + 15K} R\right)$$

Figure 4-35: Wheatstone-bridge op-amp circuit.



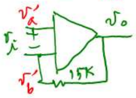
$$v_i = v_s - v_o$$
$$v_o = A v_i = A(v_s - v_o)$$
$$(1 + A) v_o = A v_s$$

$$\frac{v_o}{v_s} = \frac{A}{1 + A}$$

As $A \to \infty$

$$\frac{v_o}{v_s} = 1$$

$$A\left[\underbrace{v_a \frac{15K}{15K+R}}_{v_a'} - \underbrace{\left(v_b + \frac{v_o - v_b}{R+15K} R\right)}_{v_b'}\right] = v_o$$



$$A\left[\frac{15K}{15K+R} v_a - \left(\underbrace{1-\frac{R}{R+15K}}_{=\frac{15K}{R+15K}}\right) v_b\right] = \left[1+\frac{AR}{R+15K}\right] v_o$$

$$A\,\frac{15K}{R+15K}\left[v_a - v_b\right] = \left(1 + A\,\frac{R}{R+15K}\right) v_o$$

As $A \to \infty$

$$\boxed{v_o = \frac{15K}{R}\left(v_a - v_b\right)}$$

where $v_b = \frac{1}{2}$, $v_a = \frac{R}{R+R_s} \times 1$

when $\boxed{R_s = R}$ then $v_a = \frac{1}{2}$ + $v_a - v_b = 0 \Rightarrow \boxed{v_o = 0}$

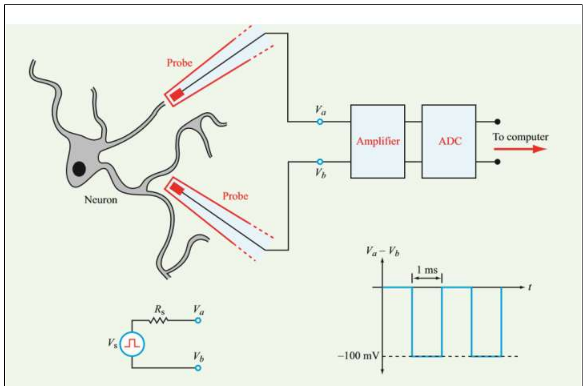
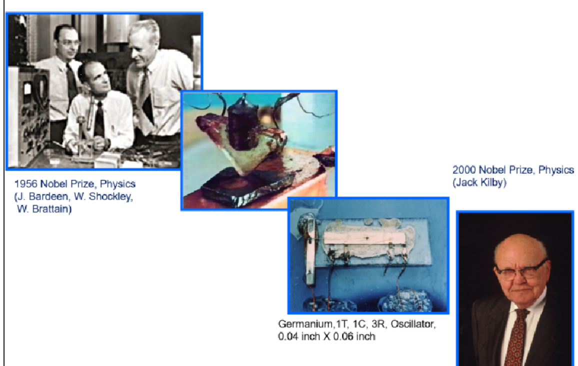


Figure 4-34: Neural-probe circuit for Example 4-11.

Transistor Invention (1948) to Integrated Circuit (1958)



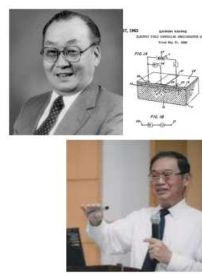
1956 Nobel Prize, Physics
(J. Bardeen, W. Shockley, W. Brattain)

2000 Nobel Prize, Physics
(Jack Kilby)

Germanium, 1T, 1C, 3R, Oscillator, 0.04 inch X 0.06 inch

First Successful Operation of MOS Transistor

Dawon Kahng (May 4, 1931- May 13, 1992)



Simon Sze

- SNU (BS), Ohio State Univ. (Ph.D. 1959)
- Dr. Kahng, with M. Atalla, fabricated a MOSFET using a gate insulator formed from high quality SiO_2 grown by a new high-pressure steam oxidation process at Bell Labs (1960)
- First successful demonstration of MOSFET was a major milestone in semiconductor technology
- Invented in 1967 a field effect memory, the first nonvolatile silicon memory (floating gate memory)
- Became Founding President of NEC, Princeton, NJ in 1988

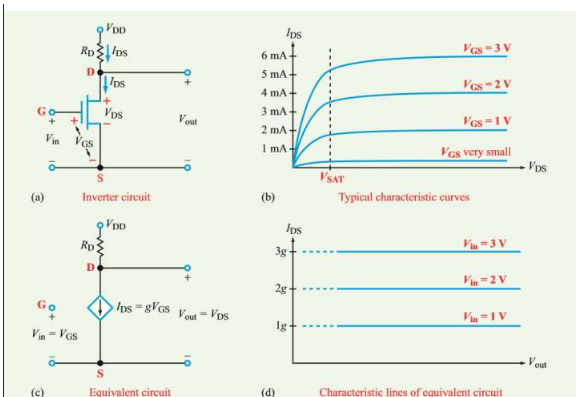


Figure 4-28: MOSFET (a) circuit, (b) characteristic curves, (c) equivalent circuit, and (d) associated characteristic lines.

4-11 The MOSFET as a Voltage-Controlled Current Source

In earlier sections, we demonstrated how op amps can be used to build buffers and amplifiers. We now examine how to realize the same outcome using MOSFETs. The simplest model of a ***MOSFET***, which stands for ***metal-oxide semiconductor field-effect transistor***, is shown in **Fig. 4-27(a)**. The vast majority of commercial computer processors are built with MOSFETs; as mentioned in Technology Brief 1 on nanotechnology, a 2010 Intel Core processor contains over 1 billion independent MOSFETs. A MOSFET has three terminals: the ***gate*** (G), the ***source*** (S), and the ***drain*** (D). Actually, it has a fourth terminal, namely its body (B), but we will ignore it for now because for many applications it is simply connected to the ground terminal. The circuit symbol for the MOSFET may look somewhat unusual, but it is actually a stylized depiction of the physical cross section of a real MOSFET. In a real MOSFET, the gate

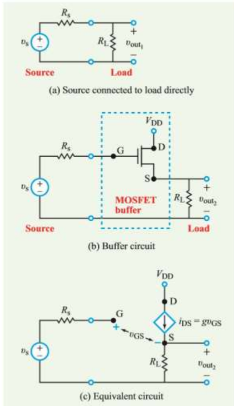


Figure 4-32: Buffer circuit for Example 4-10.

(b) With MOSFET Buffer

For the circuit in **Fig. 4-32(c)**, in which the MOSFET has been replaced with its equivalent circuit, KVL gives

$$-v_s + v_{GS} + v_{out_2} = 0.$$

Also,

$$v_{out_2} = I_{DS} R_L$$
$$= g R_L v_{GS}. = g R_L (v_s - v_{out2})$$

Simultaneous solution of the two equations gives

$$\Rightarrow \quad v_{out_2} = \left(\frac{gR_L}{1 + gR_L}\right) v_s.$$

With $g = 10$ A/V and in order for v_{out_2} to be no less than $0.99 v_s$, it is necessary that

$$R_L \geq 9.9\ \Omega,$$

which is three orders of magnitude smaller than the requirement for the unbuffered circuit.

Section 4-11: MOSFET

4.59 In Example 4-9, we analyzed a common-source amplifier without a load resistance. Consider the amplifier in **Fig. P4.59**; it is identical to the circuit in **Fig. 4-31**, except that we have added a load resistor R_L. Obtain an expression for v_{out} as a function of v_s.

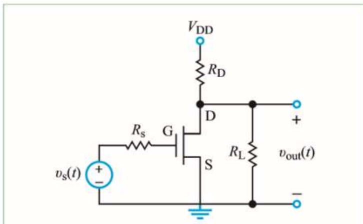
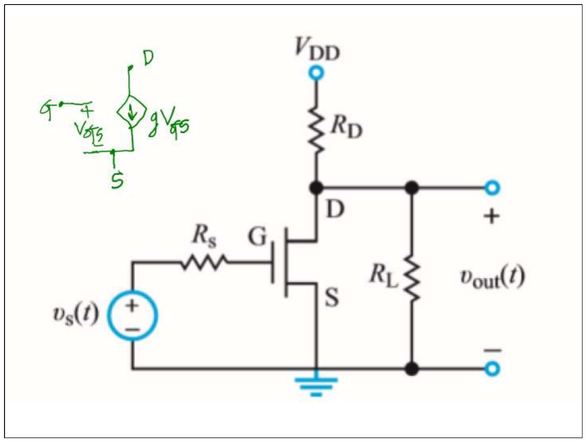


Figure P4.59: MOSFET circuit for Problem 4.59.



***4.60** Determine $v_{out}(t)$ as a function of $v_s(t)$ for the circuit in **Fig. P4.60**. Assume $V_{DD} = 2.5$ V.

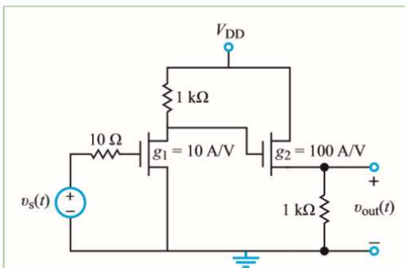
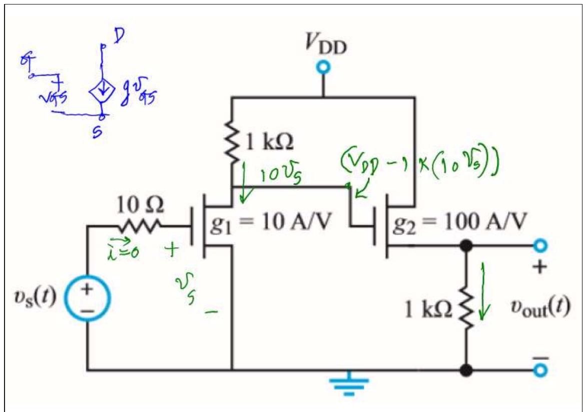
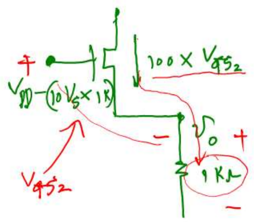


Figure P4.60: Two-MOSFET circuit for Problem 4.60.



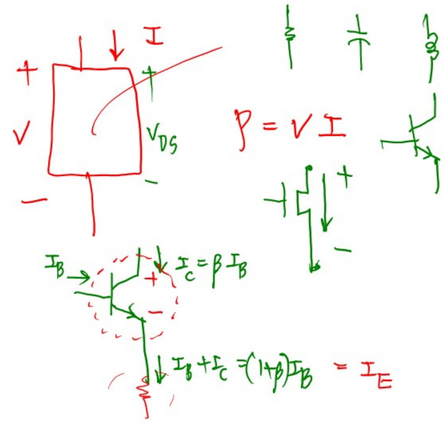


$V_{GS2} = [V_{DD} - (10 V_S \times 1K)] - v_o$

$v_o = 1\,K\Omega \times 100 \times [V_{DD} - (10 V_S \times 1K) - v_o]$

$v_o [1 + \boxed{}] = 1\,K\Omega \times 100 \times V_{DD}$

$$v_o = \frac{1\,K\Omega \times 100 \times V_{DD}}{1 + \boxed{}}$$



RC and RL First-Order Circuits

Contents

Objectives

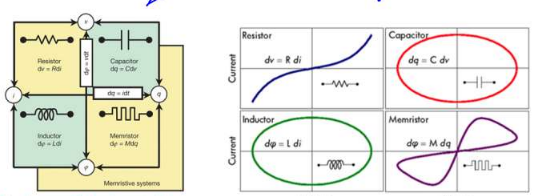
Learn to:

- Use mathematical functions to describe several types of nonperiodic waveforms.
- Define the electrical properties of a capacitor, including its i-υ relationship and energy equation.
- Combine multiple capacitors when connected in series or in parallel.
- Define the electrical properties of an inductor, including its i-υ relationship and energy equation.
- Combine multiple inductors when connected in series or in parallel.
- Analyze the transient responses of RC and RL circuits.
- Design RC op-amp circuits to perform differentiation and integration and related operations.
- Apply Multisim to analyze RC and RL circuits.

Capacitors (C) and inductors (L) are ***energy storage devices***, in contrast with resistors, which are energy dissipation devices. This chapter examines the behavior of ***RC and RL circuits***, to be followed in Chapter 6 with an examination of RLC circuits.

Capacitor $dq = C\,dv$

Linear case $q = C v$



Memristor as 4th fundamental device

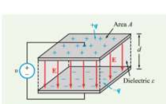


Figure 5-11: Parallel-plate capacitor with plates of area A, separated by a distance d, and filled with an insulating dielectric material of permittivity ε.

Table 5-2: Relative electrical permittivity of common insulators: $\varepsilon_r = \varepsilon/\varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

Material	Relative Permittivity ε_r
Air (at sea level)	1.0006
Teflon	2.1
Polystyrene	2.6
Paper	2–4
Glass	4.5–10
Quartz	3.8–5
Bakelite	5
Mica	5.4–6
Porcelain	5.7

$\epsilon_0 = 8.854 \times 10^{-12}$ [F/m]

SiO_2 has ϵ_r of 3.9

$\epsilon_{SiO_2} = 3.9 \times 8.854 \times 10^{-14}$ [F/m] $= 35.4 \times 10^{-14}$ [F/m]

5-2 Capacitors

► When separated by an insulating medium, any two conducting bodies (regardless of their shapes and sizes) form a ***capacitor***. A capacitor can store electric charge. ◄

The ***parallel-plate capacitor*** shown in Fig. 5-11 represents a simple configuration in which two identical conducting plates (each of area A) are separated by a distance d containing an insulating (dielectric) material of ***electrical permittivity*** ε. The permittivity of a material is usually referenced to that of free space, namely $\varepsilon_0 = 8.85 \times 10^{-12}$ farads/m (F/m). Hence, the ***relative permittivity*** of a material is defined as

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}. \quad (5.15)$$

When a dielectric material is subjected to an electric field, its atoms become partially polarized; i.e., the atom is rearranged into positive and negative domains.. The ***electric field*** E induced in the space between the conducting plates is the result of the voltage υ applied across the plates. The ***electrical susceptibility*** χ_e of a material is a measure of how susceptible that material is to electrical polarization. The permittivity ε and susceptibility χ_e are related by

$$\varepsilon = \varepsilon_0(1 + \chi_e). \quad (5.16)$$

In view of Eq. (5.15), the relative permittivity ε_r is given by

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi_e. \quad (5.17)$$

$$\boxed{q = C v \qquad v = \frac{q}{C}}$$

Free space contains no atoms; hence, its $\chi_e = 0$ and $\varepsilon_r = 1$. For air at sea level, $\varepsilon_r = 1.0006 \approx 1.0$. Table 5-2 provides typical values of ε_r for common types of insulators.

Returning to the parallel-plate capacitor, if a voltage source is connected across the two plates, as shown in Fig. 5-11, charge of equal and opposite polarity is transferred to the conducting surfaces. The plate connected to the (+) terminal of the voltage source will accumulate charge $+q$, and charge $-q$ will accumulate on the other plate. The charges induce a nearly uniform electric field E in the dielectric medium, given by

$$E = \frac{q}{\varepsilon A}, \quad (5.18)$$

with the direction of E being from the plate with $+q$ to the plate with $-q$. Moreover, E, whose unit is V/m, is related to the voltage υ through

$$E = \frac{\upsilon}{d} \quad \text{(V/m)} \quad \textbf{(parallel-plate capacitor).} \quad (5.19)$$

► For any capacitor, its ***capacitance*** C, measured in farads (F), is defined as the amount of charge q that its positive-polarity plate holds, normalized to the applied voltage responsible for that charge accumulation. ◄

Thus,

$$C = \frac{q}{\upsilon} \quad \text{(F)} \quad \textbf{(any capacitor).} \quad (5.20)$$

$$\boxed{C = \epsilon \frac{A}{d}}$$

1 cm, 1 cm, 1 μm, SiO_2

$$C = \frac{35.4 \times 10^{-14}\ \text{F/m} \times (10^{-2})^2\ m^2}{1 \times 10^{-6}\ m} = 35.4 \times 10^{-12}\ F = 35.4\ pF$$

For the parallel-plate capacitor, combining Eqs. (5.18) and (5.19) leads to $q = \varepsilon A \upsilon/d$. Upon inserting this expression for q in Eq. (5.20), we have

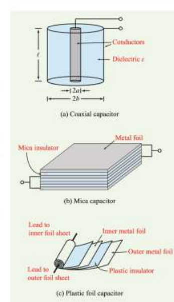
$$C = \frac{\varepsilon A}{d} \quad \textbf{(parallel-plate capacitor).} \quad (5.21)$$

Even though the expression given by Eq. (5.21) is specific to the parallel-plate capacitor, the general tenor of the expression holds true for other geometrical configurations as well. In general, the capacitance C of any two-conductor system increases with the area of the conducting surfaces, decreases with the separation between them, and is directly proportional to ε of the insulating material. For example, the capacitance of a ***coaxial capacitor*** consisting of two concentric conducting cylinders of radii a and b (Fig. 5-12(a)) and separated by a dielectric material of permittivity ε is given by

$$C = \frac{2\pi\varepsilon\ell}{\ln(b/a)} \quad \textbf{(coaxial capacitor),} \quad (5.22)$$

where ℓ is the length of the capacitor and $\ln(b/a)$ is the natural logarithm of (b/a). The spacing between the cylinders is $(b-a)$; reducing this spacing, while holding b constant, requires reducing the ratio (b/a), which reduces the value of $\ln(b/a)$, thereby increasing the magnitude of C.

The ***mica capacitor*** shown in Fig. 5-12(b) consists of a stack of conducting plates, interleaved by sheets of mica (dielectric). The ***plastic-foil capacitor*** in Fig. 5-12(c) is constructed by rolling flexible conducting foils (separated by a plastic layer) into a spindle-like configuration. Small capacitors used in microcircuits typically have capacitances in the picofarad (10^{-12} F) to microfarad (10^{-6} F) range. Large capacitors used in power-transmission substations may have capacitors in the range of millifarads (10^{-3} F). Using thin-film polymers for the dielectric insulator and carbon nanotubes for the electrodes (terminals), a new type of capacitor (sometimes called a ***supercapacitor*** or ***nanocapacitor***) was developed in the 1990s with the express goal of significantly increasing the amount of charge that the conductors can hold (at a specified voltage level). Such capacitors have capacitance values that are several orders of magnitude greater than conventional capacitors of comparable size. The new fabrication techniques have not only expanded the versatility of capacitors in electronic circuits, but they have also introduced the use of supercapacitors as energy-storage devices in many electronic applications (see Technology Brief 12: Supercapacitors).



(a) Coaxial capacitor

(b) Mica capacitor

(c) Plastic foil capacitor

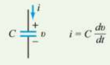
Figure 5-12: Various types of capacitors.

5-2.1 Electrical Properties of Capacitors

According to Eq. (5.20), $q = C\upsilon$. Application of the standard definition for current (Eq. (1.3)) provides the expression for the current i through a capacitor as

$$\boxed{i = \frac{dq}{dt} = C\frac{d\upsilon}{dt},} \quad (5.23)$$

where the direction of i and the polarity of υ are defined in accordance with the passive sign convention (Fig. 5-13).



$$i = C\,\frac{dv}{dt}$$

Figure 5-13: Passive sign convention for capacitor: if current i is entering the (+) voltage terminal across the capacitor, then power is getting transferred into the capacitor. Conversely, if i is leaving the (+) terminal, then power is getting released from the capacitor.

The i–v relationship expressed by Eq. (5.23) conveys a very important condition, namely:

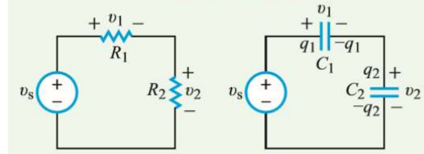
► The voltage across a capacitor cannot change instantaneously, but the current can. ◄

This assertion is supported by the observation that if v were to change values in zero time, dv/dt would be infinite, as a result of which the current i would be also infinite. Since i cannot be infinite, v cannot change instantaneously.

Another attribute of Eq. (5.23) relates to the behavior of a capacitor under dc conditions (constant voltage across it). Since $dv/dt = 0$ for a dc voltage, it follows that $i = 0$. Such a behavior is characteristic of an open circuit, through which no current flows even when a non-zero voltage exists across it. Thus:

► Under dc conditions, a capacitor behaves like an open circuit. ◄

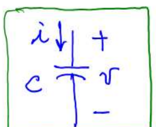
Voltage Division



(a) $v_1 = \left(\dfrac{R_1}{R_1 + R_2}\right) v_s$ (b) $v_1 = \left(\dfrac{C_2}{C_1 + C_2}\right) v_s$

$v_2 = \left(\dfrac{R_2}{R_1 + R_2}\right) v_s$ $v_2 = \left(\dfrac{C_1}{C_1 + C_2}\right) v_s$

Figure 5-19: Voltage-division rules for (a) in-series resistors and (b) in-series capacitors.



$$q(t) = C\,v(t)$$

$$i(t) = C\,\frac{dv(t)}{dt}$$

$$q(t) = \int_{-\infty}^{t} i(\tau)\,d\tau = \int_{-\infty}^{0} i(\tau)\,d\tau + \int_{0}^{t} i(\tau)\,d\tau = q(0) + \int_0^t i(\tau)\,d\tau$$

$$\frac{dq(t)}{dt} = \underbrace{\frac{dq(0)}{dt}}_{=0} + i(t)$$

$$\frac{d}{dt}\big(C\,v(t)\big) = \boxed{C\,\frac{dv(t)}{dt} = i(t)}$$

$$p(t) = v(t)\,i(t) = v(t)\,C\,\frac{dv(t)}{dt} = C\,v(t)\,\frac{dv(t)}{dt}$$

$$E(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = C\int_{v(-\infty)}^{v(t)} v(\tau)\,\frac{dv(\tau)}{\cancel{d\tau}}\,\cancel{d\tau} = \frac{1}{2}C\,v^2(t), \quad v(-\infty) = 0$$

Derive the equivalent voltage-division equation for the series capacitors C_1 and C_2 in Fig. 5-19(b). Assume that the capacitors had no charge on them before they were connected to v_s.

Solution: From the standpoint of the source v_s, it "sees" an equivalent, single capacitor C given by the series combination of C_1 and C_2, namely

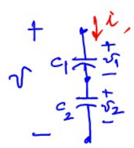
$$C = \frac{C_1 C_2}{C_1 + C_2}. \tag{5.44}$$

The voltage across C is v_s. The law of conservation of energy requires that the energy that would be stored in the equivalent capacitor C be equal to the sum of the energies stored in C_1 and C_2. Hence, application of Eq. (5.29) gives

$$\frac{1}{2}\,C v_s^2 = \frac{1}{2}\,C_1 v_1^2 + \frac{1}{2}\,C_2 v_2^2. \tag{5.45}$$

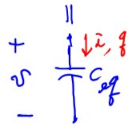
Upon replacing C with the expression given by Eq. (5.44) and replacing the source voltage with $v_s = v_1 + v_2$, we have

$$\frac{1}{2}\left(\frac{C_1 C_2}{C_1 + C_2}\right)(v_1 + v_2)^2 = \frac{1}{2}\,C_1 v_1^2 + \frac{1}{2}\,C_2 v_2^2, \tag{5.46}$$



$$v = v_1 + v_2 = \frac{q}{C_1} + \frac{q}{C_2}$$

||

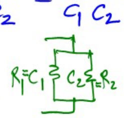


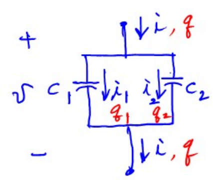
$$v = \frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

same as conductance (G) connection

$$\Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

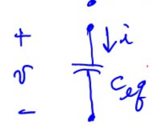




$$i = i_1 + i_2$$

$$q = q_1 + q_2$$

$$\frac{dq}{dt} = \frac{dq_1}{dt} + \frac{dq_2}{dt}$$



$$\frac{dq}{dt} = \frac{d}{dt}\left(C_{eq}\,v\right)$$

$$= C_{eq}\frac{dv}{dt} = \frac{d}{dt}(C_1 v) + \frac{d}{dt}(C_2 v) = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} = (C_1 + C_2)\frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2}$$

Exercise 5-9: Determine C_{eq} and $v_{eq}(0)$ at terminals (a, b) for the circuit in **Fig. E5.9** given that $C_1 = 6\ \mu$F, $C_2 = 4\ \mu$F, $C_3 = 8\ \mu$F, and the initial voltages on the three capacitors are $v_1(0) = 5$ V and $v_2(0) = v_3(0) = 10$ V, respectively.

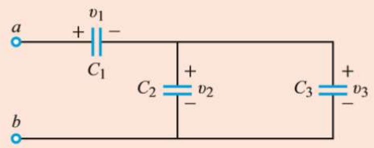


Figure E5.9

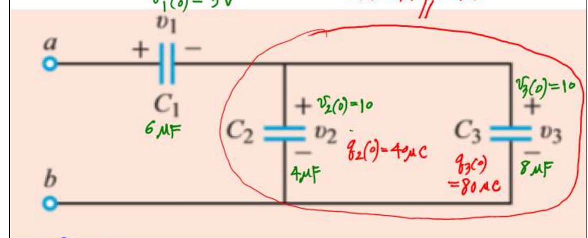
Answer: $C_{eq} = 4\ \mu$F, $v_{eq}(0) = 15$ V. (See CAD)

$q_1(0) = 30\,\mu C$

$v_1(0) = 5V$

$q_{23}(0) = 120\,\mu C$

$4\mu F + 8\mu F = 12\mu F$



$v_{ab}(0) = 5V + 10V = 15V$

$C_{ab} = \frac{6\mu F(12\mu F)}{6\mu F + 12\mu F} = \frac{72(\mu F)^2}{18\mu F} = 4\mu F$

5.18 Reduce the circuit in **Fig. P5.18** into a single equivalent capacitor at terminals (a, b). Assume that all initial voltages are zero at $t = 0$.

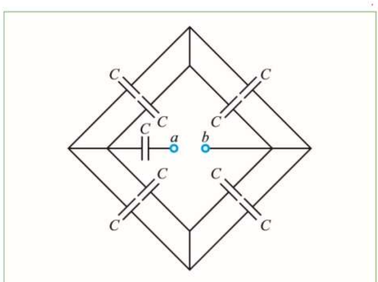
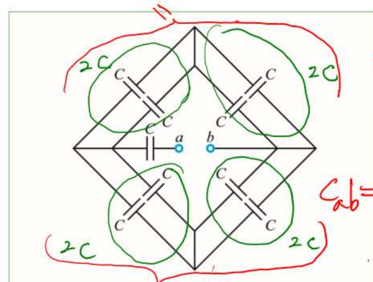


Figure P5.18: Circuit for Problem 5.18.

5.18 Reduce the circuit in **Fig. P5.18** into a single equivalent capacitor at terminals (a, b). Assume that all initial voltages are zero at $t = 0$.



$C_{ab} = \frac{C \cdot 2C}{C + 2C} = \frac{2}{3}C$

Figure P5.18: Circuit for Problem 5.18.

***5.19** For the circuit in **Fig. P5.19**, find C_{eq} at terminals (a, b). Assume all initial voltages to be zero.

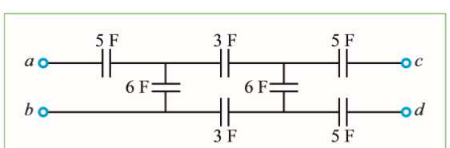


Figure P5.19: Circuit for Problems 5.19 and 5.20.

***5.19** For the circuit in **Fig. P5.19**, find C_{eq} at terminals (a, b) (c, d). Assume all initial voltages to be zero.

$6F + 1.2F = 7.2F$

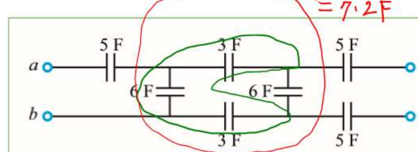


Figure P5.19: Circuit for Problems 5.19 and 5.20.

$\frac{1}{C_{cd}} = \frac{1}{5} + \frac{1}{7.2} + \frac{1}{5} = \frac{7.2 \times 2 + 5}{36} = \frac{19.4}{36}$

$\Downarrow$

$\boxed{C_{cd} = \frac{36}{19.4}}$

$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{2+1+2}{6} = \frac{5}{6}$

$C_{eq} = \frac{6}{5}$ F

5-3 Inductors

Any current-carrying conductor, whether straight or coiled, forms an inductor. A current produces a magnetic field, which stores energy that can be released later in the form of another current. Also, since every wire acts like an inductor, we have small amounts of stray inductance in every circuit. Fortunately, this can be ignored except at extremely high frequencies (microwave band).

Inductors exhibit a number of useful properties, including magnetic coupling and electromagnetic induction. They are employed in microphones and loudspeakers, magnetic relays and sensors, theft detection devices, and motors and generators, and they provide wireless power transmission and data communication (albeit over relatively short distances).

► Capacitors and inductors constitute a canonical pair of devices. Whereas a capacitor can store energy through the electric field induced by the voltage imposed across its terminals, an inductor can store magnetic energy through the magnetic field induced by the current flowing through its wires. ◄

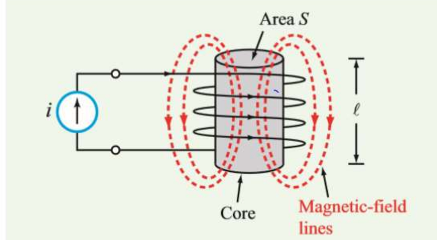


Figure 5-20: The inductance of a solenoid of length ℓ and cross-sectional area S is $L = \mu N^2 S/\ell$, where N is the number of turns and μ is the magnetic permeability of the core material.

$\phi \overset{}{\underset{L}{\longleftrightarrow}} i$

$$\phi = L\, i$$

$$\frac{d\phi}{dt} = L\,\frac{di}{dt}$$

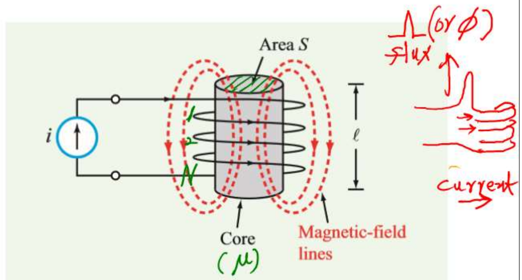


Figure 5-20: The inductance of a solenoid of length ℓ and cross-sectional area S is $\boxed{L = \mu N^2 S/\ell}$, where N is the number of turns and μ is the magnetic permeability of the core material.

5-3.1 Electrical Properties

According to Faraday's law, if the magnetic-flux linkage in an inductor (or circuit) changes with time, it induces a voltage υ across the inductor's terminals given by

$$\upsilon = \frac{d\Lambda}{dt}. \qquad (5.54)$$

$$v = \frac{d\phi}{dt}$$

In view of Eq. (5.51),

$$\upsilon = \frac{d}{dt}\,(Li) = L\,\frac{di}{dt}. \qquad (5.55)$$

This i–υ relationship adheres to the passive sign convention introduced earlier for resistors and capacitors. If the direction of i is into the (+) voltage terminal of the inductor (**Fig. 5-22**), then the inductor is receiving power. Also, the same logic that led us earlier to the conclusion that the voltage across a capacitor cannot change instantaneously leads us now to the conclusion:

► The current through an inductor cannot change instantaneously, but the voltage can. ◄

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt', \qquad (5.56)$$

where t_0 is an initial reference point in time.

The power delivered to the inductor is given by

$$p(t) = \upsilon i = Li\,\frac{di}{dt}, \qquad (5.57)$$

and as with the resistor and the capacitor, the sign of p determines whether the inductor is receiving power ($p > 0$) or delivering it ($p < 0$). The accumulation of power over time constitutes the storage of energy. The magnetic energy stored in an inductor is

$$w(t) = \int_{-\infty}^{t} p\, dt' = \int_{-\infty}^{t} \left(Li\,\frac{di}{dt'}\right) dt', \qquad (5.58) \quad = L\int_{i(-\infty)}^{i(t)} i\, di$$

which yields

$$w(t) = \frac{1}{2}\,L\,i^2(t) \qquad \text{(J)}, \qquad (5.59) \quad = \frac{1}{2}L\,i^2(t)$$

same as resistor connection

Combining In-Series Inductors

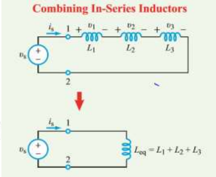
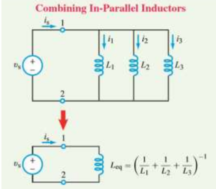


Figure 5-24: Inductors in series.

Combining In-Parallel Inductors



*5.31 The values of all inductors in the circuit of Fig. P5.31 are in millihenrys. Determine L_{eq}.

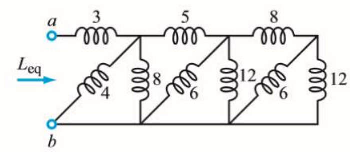


Figure P5.31: Circuit for Problem 5.31.

*5.31 The values of all inductors in the circuit of Fig. P5.31 are in millihenrys. Determine L_{eq}.

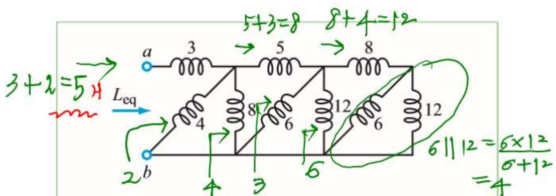


Figure P5.31: Circuit for Problem 5.31.

*5.15 Determine voltages v_1 to v_3 in the circuit of Fig. P5.15 under dc conditions.

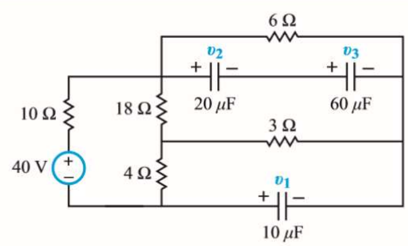


Figure P5.15: Circuit for Problem 5.15.

*5.15 Determine voltages v_1 to v_3 in the circuit of Fig. P5.15 under dc conditions.

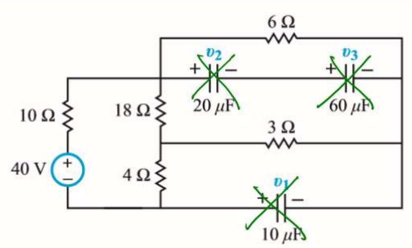


Figure P5.15: Circuit for Problem 5.15.

5.28 For the circuit in Fig. P5.28, determine the voltage across C and the currents through L_1 and L_2 under dc conditions.

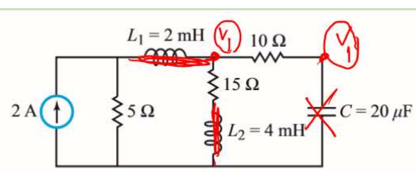


Figure P5.28: Circuit for Problem 5.28.

*5.29 For the circuit in Fig. P5.29, determine the voltages across C_1 and C_2 and the currents through L_1 and L_2 under dc conditions.

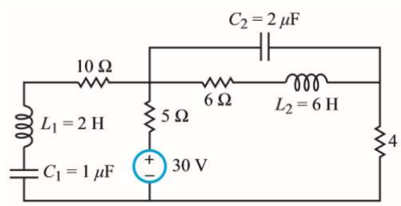


Figure P5.29: Circuit for Problem 5.29.

5.16 Determine the voltages across the two capacitors in the circuit of Fig. P5.16 under dc conditions.

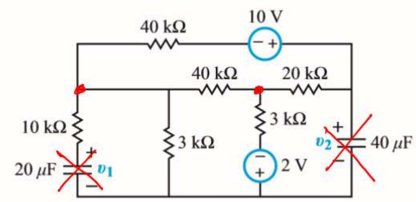


Figure P5.16: Circuit for Problem 5.16.

5.16 Determine the voltages across the two capacitors in the circuit of Fig. P5.16 under dc conditions.

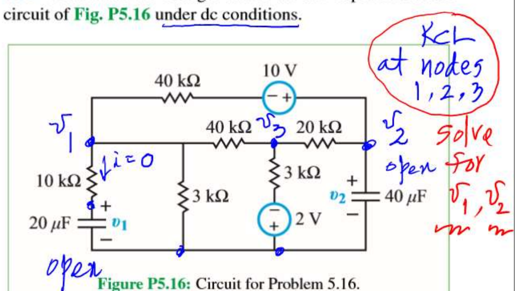
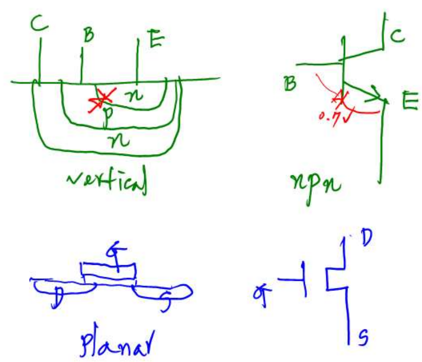
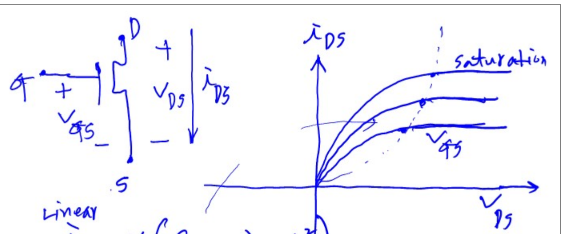


Figure P5.16: Circuit for Problem 5.16.





Linear

$$i_{DS} = K\left((v_{GS}-V_{th})v_{DS}-\frac{1}{2}v_{DS}^2\right) \quad v_{DS} = v_{GS}-V_{th}$$

Sat.

$$i_{DS} = \frac{K}{2}\left(v_{GS}-V_{th}\right)^2$$